

Target Evasion Strategy Against a Finite Set of Missile Guidance Laws

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Abstract—In this paper, a multiple model adaptive evasion strategy for a target aircraft from a homing missile employing a linear guidance law is proposed. We assume arbitrary-order linear missile and target dynamics, bounded target control, nonlinear kinematics, and the missile employing one of a finite set of possible guidance laws. Specific cases are numerically analyzed in which the attacking missile uses proportional navigation, augmented proportional navigation, or optimal guidance.

I. INTRODUCTION

Guidance laws for intercepting a moving target, such as aircraft in this study, have traditionally been developed for one-on-one engagements, assuming perfect information and linearized kinematics [1]. In order to respond to a threat from a homing missile employing such guidance laws, the target can perform an evasive maneuver, which can be either arbitrary or optimally adjusted against the incoming missile. It is possible to develop an optimal evasion strategy using optimal control theory tools, however it requires information on the missile's future behavior, i.e., its guidance law and parameters. A case study where such a problem was formulated as a one-sided optimal control problem against a PN-guided missile were presented in [2]–[4].

Although there has been substantial work in the literature on target evasion, most of the research concentrated on PN-guided missiles, which leaves the strategies against other guidance laws lacking. In a recent work [5], optimal evasion strategy for a target aircraft from a homing missile employing a linear guidance law has been derived. The problem was analyzed for arbitrary-order adversaries dynamics, bounded target controls, and assuming perfect information. The underlying assumption in this solution is that the missile's guidance strategy is exactly known to the target.

In this paper, an optimal multiple model adaptive evasion strategy (MMAES) is proposed which greatly relaxes the assumptions made in [5]. We assume that the missile is chasing the target using one of a closed set of possible linear guidance laws and guidance parameters. An EKF-based static multiple model online identification scheme is used to identify the active missile guidance strategy. For each such a guidance strategy, an optimal target evasion law is paired. This law is derived based on a linearized model, but implemented within the nonlinear setting. The final target

evasive maneuver command is computed using a multiple model adaptive control (MMAC) framework.

From implementation point of view, we discuss the observability issues if only the line of sight (LOS) angle is measured. Considering different sets of noisy measurements and two fusing criteria for the MMAC approach, the performance of the proposed MMAES is compared through extensive simulations to the scenario when the target has perfect information about the attacking missile.

II. MATHEMATICAL MODELS

In this section we present the full nonlinear kinematics and dynamics equations of the missile-target evasion problem, serving for analysis. Then, linearized equations, used for the derivation of the optimal MMAES, are presented.

A. Nonlinear Kinematics and Dynamics

In Figure 1, the planar point mass missile-target engagement geometry is shown. The speed, acceleration, and flight-path angles are denoted by V , a , and γ , respectively; the range between the missile and target is r , and λ is the angle between the LOS and X_I axis. The acceleration vector of the missile a_M and the target a_T are assumed to be perpendicular to their own velocity vectors V_M and V_T , respectively.

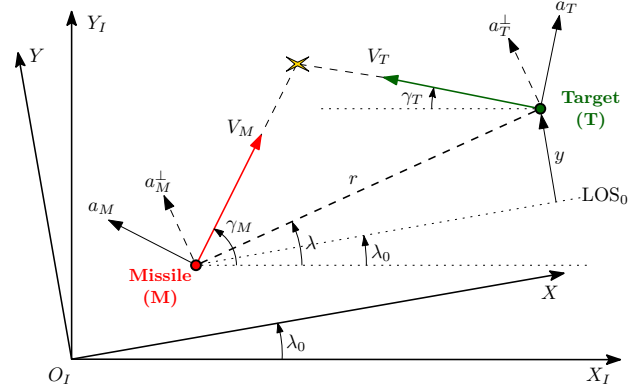


Fig. 1. Planar missile-target engagement geometry.

The engagement kinematics, expressed in a polar coordinate system (r, λ) attached to the missile, is

$$\dot{r} = -(V_M \cos(\gamma_M - \lambda) + V_T \cos(\gamma_T + \lambda)) \triangleq V_r \quad (1)$$

$$\dot{\lambda} = -(V_M \sin(\gamma_M - \lambda) - V_T \sin(\gamma_T + \lambda)) / r \triangleq V_\lambda / r \quad (2)$$

We denote the running time as t . The endgame initiates at $t = 0$ with $\dot{r}(t = 0) < 0$ and terminates at $t = t_f$, where

$$t_f = \arg \inf_t \{r(t)\dot{r}(t) = 0\}, \quad t > 0, \quad (3)$$

allows to define the time-to-go, t_{go} , by $t_{go} \triangleq t_f - t$.

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The missile-target separation at $t = t_f$ is called the miss distance, i.e., $\text{Miss} = r(t_f)$. During the endgame, the missile and the target are assumed to move at a constant speeds. We assume arbitrary-order linear dynamics for both the missile and the target, defined as follows

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x}_i + \mathbf{B}_i u_i \\ a_i = \mathbf{C}_i \mathbf{x}_i + d_i u_i, \quad i = \{M, T\}, \\ \dot{\gamma}_i = a_i/V_i \end{cases} \quad (4)$$

where $\mathbf{x}_i \in \mathbb{R}^{n_i}$ is the state vector of an entity's internal state variables, a_i and u_i are entity's acceleration and acceleration command, respectively. We also assume that $|u_T| \leq a_T^{\text{max}}$.

B. Linearized Equations of Motion

If during the endgame the missile and target deviations from the collision triangle are small, then the linearization around the initial LOS is justified [1]. In Figure 1, the X -axis, aligned with the LOS used for linearization, is denoted as LOS_0 . The relative displacement between the target and missile normal to this direction is y . Under linearization assumption, the missile and target accelerations normal to LOS_0 , i.e., a_M^\perp and a_T^\perp , can be approximated by

$$a_M^\perp \approx k_M a_M, \quad k_M = \cos(\gamma_{M0} - \lambda_0), \quad (5a)$$

$$a_T^\perp \approx k_T a_T, \quad k_T = \cos(\gamma_{T0} + \lambda_0), \quad (5b)$$

where the subscript "0" denotes the initial value around which linearization has been performed. It is assumed that $|\gamma_{M0} - \lambda_0| < \pi/2$ and $|\gamma_{T0} + \lambda_0| < \pi/2$.

As soon as the collision triangle is reached and maintained, the speed V_r is constant and the interception time t_f can be assumed fixed and approximated by

$$\tilde{t}_f \approx -r_0/V_r. \quad (6)$$

Let's define the state vector of the linearized problem as

$$\mathbf{x}^l = [x_1 \quad x_2 \quad \mathbf{x}_M^T \quad \mathbf{x}_T^T]^T \triangleq [y \quad \dot{y} \quad \mathbf{x}_M^T \quad \mathbf{x}_T^T]^T.$$

Then, the missile-target equations of relative motion normal to LOS_0 can be expressed as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = k_T a_T - k_M a_M \\ \dot{\mathbf{x}}_M = \mathbf{A}_M \mathbf{x}_M + \mathbf{B}_M u_M \\ \dot{\mathbf{x}}_T = \mathbf{A}_T \mathbf{x}_T + \mathbf{B}_T u_T \end{cases} \quad (7)$$

Using (4) and (5), the above equations can be rewritten as

$$\dot{\mathbf{x}}^l = \mathbf{A} \mathbf{x}^l + \mathbf{B} u_T^\perp + \mathbf{C} u_M^\perp, \quad (8)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & -\tilde{\mathbf{C}}_M & \tilde{\mathbf{C}}_T \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_M & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_T \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ d_T \\ \mathbf{0} \\ \tilde{\mathbf{B}}_T \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 \\ -d_M \\ \tilde{\mathbf{B}}_M \\ \mathbf{0} \end{bmatrix},$$

where $\tilde{\mathbf{B}}_M = k_i^{-1} \mathbf{B}_i$, $\tilde{\mathbf{C}}_i = k_M \mathbf{C}_i$, $\forall i \in \{M, T\}$, u_M^\perp and u_T^\perp being, respectively, the missile and target acceleration commands normal to LOS_0 .

III. OPTIMAL TARGET EVASION STRATEGY

In this section, a nonlinear implementation of the optimal (linear) target evasion strategy [5] from a missile employing a linear guidance law is presented.

A. Missile Guidance

In this paper, we consider a large family of linear missile guidance laws, which all have the same linear form

$$u_M^\perp = \mathbf{K}(t_{go}) \mathbf{x}^l + K_{u_T}(t_{go}) u_T^\perp, \quad (9)$$

where

$$\mathbf{K}(t_{go}) = [K_1 \quad K_2 \quad \mathbf{K}_M \quad \mathbf{K}_T].$$

In this study, we assume that the missile has perfect information about its own and target's states.

B. Optimal Evasion Problem - Linear Setting

Using the equations of motion (EOM) of the linearized missile-target engagement (8) together with (9), we obtain the EOM of the one-sided evasion problem

$$\dot{\mathbf{x}}^l = \mathbf{A}_E(t_{go}) \mathbf{x}^l + \mathbf{B}_E(t_{go}) u_T^\perp, \quad (10)$$

$$\begin{aligned} \mathbf{A}_E(t_{go}) = & \begin{bmatrix} 0 & 1 & \mathbf{0} & \mathbf{0} \\ -d_M K_1 & -d_M K_2 & -\tilde{\mathbf{C}}_M - d_M \mathbf{K}_M & \tilde{\mathbf{C}}_T - d_M \mathbf{K}_T \\ \tilde{\mathbf{B}}_M K_1 & \tilde{\mathbf{B}}_M K_2 & \mathbf{A}_M + \tilde{\mathbf{B}}_M \mathbf{K}_M & \tilde{\mathbf{B}}_M \mathbf{K}_T \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_T \end{bmatrix}, \\ \mathbf{B}_E(t_{go}) = & \begin{bmatrix} 0 & d_T - d_M K_{u_T} & \tilde{\mathbf{B}}_M^T K_{u_T} & \tilde{\mathbf{B}}_T^T \end{bmatrix}^T. \end{aligned}$$

Theorem 1 ([5]): The optimal target evasion strategy from a homing missile employing a linear guidance law of the form (9), maximizing the following cost function

$$J_E = y^2(t_f)/2 \quad (11)$$

subject to the EOM of (10) and under the constraint that the target's control $|u_T| \leq a_T^{\text{max}}$ is bounded, is given by

$$u_T^{\perp*} = a_T^{\perp \text{max}} \text{sign}(s_{MT}) \text{sign}(Z_{MT}); \quad Z_{MT}(0) \neq 0, \quad (12)$$

where s_{MT} is the switching function and Z_{MT} is the well-known zero-effort-miss (ZEM) distance, i.e.,

$$s_{MT} = \mathbf{D}_E \Phi_E(t_f, t) \mathbf{B}_E(t_{go}), \quad (13)$$

$$Z_{MT} = \mathbf{D}_E \Phi_E(t_f, t) \mathbf{x}^l, \quad (14)$$

where $\mathbf{D}_E = [1 \ 0 \ 0 \ 0]$. The transition matrix $\Phi_E(t_f, t)$, associated with the solution of (10), satisfies

$$\dot{\Phi}_E(t_f, t) = -\Phi_E(t_f, t) \mathbf{A}_E(t_{go}), \quad \Phi_E(t_f, t_f) = \mathbf{I}. \quad (15)$$

In (12), $a_T^{\perp \text{max}}$ is a projection of a_T^{max} in the direction perpendicular to LOS_0 , and is given by: $a_T^{\perp \text{max}} = k_T a_T^{\text{max}}$.

If the engagement is initialized in the singular region, i.e., $Z_{MT}(0) = 0$, or if $Z_{MT}(t) = 0$, then it was suggested in [5] that for $Z_{MT}(t) = 0$ the optimal evasion strategy $u_T^{\perp*}$ should be chosen as either $u_T^{\perp \text{max}}$ or $-u_T^{\perp \text{max}}$. Moreover, it was also shown that if $u_T^{\perp*}$ is employed within the linear setting (10), the optimal ZEM dynamics \dot{Z}_{MT}^* is given by

$$\dot{Z}_{MT}^* = \text{sign}(Z_{MT}) |s_{MT}| a_T^{\perp \text{max}}. \quad (16)$$

Consequently, $|Z_{MT}|$ is a monotonically increasing function of time, satisfying $\forall t \in [0, t_f]$,

$$\text{sign}(Z_{MT}(t)) = \text{sign}(Z_{MT}(t_f)), \quad Z_{MT}(0) \neq 0. \quad (17)$$

Defining $m = \int_0^{t_f} |s_{MT}| a_T^{\perp \max} dt$, we obtain the expected miss distance (under the assumption of linearity)

$$\text{Miss}_{\text{exp}} = |y(t_f)| = |Z_{MT}(t_f)| = |Z_{MT}(0)| + m. \quad (18)$$

Note that s_{MT} converges asymptotically to zero as $t_{go} \rightarrow \infty$.

C. Nonlinear Implementation

To implement the optimal evasion strategy within the nonlinear setting, $u_T^{\perp*}$ must be projected in the direction normal to the target's velocity vector, denoted as u_T^* . Assuming small deviations from the collision triangle yields to $k_T / \cos(\gamma_T + \lambda) \approx 1$, hence u_T^* can be approximated by

$$u_T^* = \frac{u_T^{\perp*}}{\cos(\gamma_T + \lambda)} \approx a_T^{\max} \text{sign}(s_{MT}) \text{sign}(Z_{MT}) \quad (19)$$

for $Z_{MT}(0) \neq 0$. Once a collision triangle is reached and maintained, the speed V_r is constant and the approximation of the interception time \tilde{t}_f , given by (6), can be assumed fixed throughout the engagement. Thus, the relevant components of $\Phi_E(\tilde{t}_f, t)$ can be precomputed off-line.

The displacement y normal to LOS_0 can be expressed at any time by

$$y = r \sin(\lambda - \lambda_0), \quad (20)$$

Differentiating (20) with respect to time, yields

$$\dot{y} = V_r \sin(\lambda - \lambda_0) + V_\lambda \cos(\lambda - \lambda_0). \quad (21)$$

Using the above expressions for the computation of Z_{MT} replaces the dependency on y and \dot{y} by the kinematics variables V_r , V_λ , r , and λ . Moreover, assuming small deviations from the collision triangle, thus $\lambda - \lambda_0 \approx 0$, the displacement y can be assumed zero and \dot{y} can be approximated as $\dot{y} = V_\lambda$.

IV. ESTIMATOR IN THE LOOP

The assumption of Theorem 1 is that the missile's guidance is known to the target. In this section, this assumption is greatly relaxed by employing a multiple model-based online identification scheme together with the MMAC approach.

A. Static Multiple Model Estimation

In the multiple model estimation approach [6], the system operates in one of a finite number of models. The operating model is often called as "mode" or "regime" of the system. Let's assume that the guidance law of the missile u_M , being fixed throughout the engagement, is one of p possible ones

$$u_M \in \{u_M^1, \dots, u_M^p\}. \quad (22)$$

Each u_M^j is characterized by a set of five parameters, i.e., $\{K_1^j, K_2^j, \mathbf{K}_M^j, \mathbf{K}_T^j, K_{u_T}^j\}$. These parameters exclusively define the j th regime and might be functions of t_{go} . Each such regime will generate different missile acceleration commands u_M and will also result in different missile trajectories.

As the engagement model is nonlinear, a mode-matched extended Kalman filter (EKF) is used in the next [7]. The main idea is to design and run in parallel a bank of p EKFs, each matching a different possible regime j .

1) *System Model for Estimation*: It is assumed that target-related parameters \mathbf{x}_T , γ_T , and V_T are known to a very high accuracy (via some navigation system). Then, based on the constant missile speed assumption, the j th regime's dynamics is governed by the following set of equations

$$\begin{cases} \dot{r} &= V_r \\ \dot{\lambda} &= V_\lambda / r \\ \dot{\mathbf{x}}_M &= \mathbf{A}_M \mathbf{x}_M + \mathbf{B}_M u_M^j \\ \dot{\gamma}_M &= (\mathbf{C}_M \mathbf{x}_M + d_M u_M^j) / V_M \\ \dot{V}_M &= 0 \end{cases}, \quad (23)$$

where V_r and V_λ are given by (1) and (2), respectively, and u_M^j is the missile's acceleration command obeying the j th regime and being defined as

$$u_M^j = \frac{K_1^j y + K_2^j \dot{y} + \mathbf{K}_M^j \mathbf{x}_M + \mathbf{K}_T^j \mathbf{x}_T + K_{u_T}^j u_T^\perp}{\cos(\gamma_M - \lambda)}. \quad (24)$$

The variable y and \dot{y} is given by (20) and (21), respectively. The discrete-time version of (23) for u_M^j can be rewritten as

$$\mathbf{x}_k = \mathbf{f}_{k-1}^j(\mathbf{x}_{k-1}, u_T^\perp), \quad (25)$$

where $\mathbf{x}_k \triangleq [r, \lambda, \mathbf{x}_M^T, \gamma_M, V_M]^T$ is the state vector at time $t_k = kT_s$ with $\dim(\mathbf{x}_k) = n_x$, \mathbf{f}_{k-1}^j is a function derived by integrating (23) from t_{k-1} to t_k , and j is the specific regime.

2) *Measurement Model*: The target is assumed to be equipped with an electro-optic seeker and/or a radar. Thus, one may acquire: a) both r and λ measurements, or b) only λ , i.e., bearing-only measurements. The measurement vector $\mathbf{z}_k \in \mathbb{R}^{n_z}$ is assumed to be acquired at a sampling time T_s^m and being corrupted by a zero-mean mutually independent white Gaussian noise sequence $\mathbf{v}_k \in \mathbb{R}^{n_z}$. The measurement model, when all possible measurements are available, is

$$\mathbf{z}_k = \mathbf{H} \mathbf{x}_k + \mathbf{v}_k = [r_k \quad \lambda_k]^T + \mathbf{v}_k, \quad (26)$$

where $\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ with $\mathbf{R} = \text{diag}(\sigma_r^2, \sigma_\lambda^2)$, and $\mathbf{H} \in \mathbb{R}^{n_z \times n_x}$ is an appropriate measurement matrix.

3) *Mode-Matched Filtering*: For the given regime j , the time propagation step from t_{k-1} to t_k is computed using (25) and the appropriate u_M^j , i.e.,

$$\hat{\mathbf{x}}_{k|k-1}^j = \mathbf{f}_{k-1}^j(\hat{\mathbf{x}}_{k-1|k-1}, u_T^\perp). \quad (27)$$

The j th mode state transition matrix Φ_k^j associated with the system dynamics (23) is

$$\Phi_k^j = \exp(\mathbf{F}_x^j T_s), \quad (28)$$

where $T_s \triangleq t_k - t_{k-1}$ is the sampling time used for time propagation and \mathbf{F}_x^j is the associated Jacobian matrix,

$$\mathbf{F}_x^j = \partial \mathbf{f}^j / \partial \mathbf{x} |_{\mathbf{x} = \hat{\mathbf{x}}_{k-1|k-1}^j}, \quad (29)$$

assumed to be fixed for $t \in (t_{k-1}, t_k]$. The prediction error covariance matrix of the filter matched to the j th regime is

$$\mathbf{P}_{k|k-1}^j = \Phi_k^j \mathbf{P}_{k-1|k-1}^j \Phi_k^{jT} + \mathbf{Q}_k^j, \quad (30)$$

where \mathbf{Q}_k^j is an artificial process noise covariance matrix

$$\mathbf{Q}_k^j = \int_0^{T_s} \Phi_k^j(\eta) \Psi^j \Phi_k^j(\eta)^T d\eta, \quad (31)$$

used as a tuning matrix [6]. In (31), Ψ^j is a matrix which has only one nonzero element, $\Psi^j(4,4) = \psi^j$, and is used as the tuning parameter of the j th mode-matched EKF.

Using a new measurement z_k , the state estimate $\hat{x}_{k|k}^j$ and covariance $P_{k|k}^j$ of the filter matched to the j th mode are updated using the standard EKF equations

$$\nu_k^j = z_k - H\hat{x}_{k|k-1}^j, \quad (32)$$

$$\hat{x}_{k|k}^j = \hat{x}_{k|k-1}^j + K_k^j \nu_k^j, \quad (33)$$

where ν_k^j is the innovation of the j th mode-matched filter and K_k^j is the Kalman gain, computed as

$$K_k^j = P_{k|k}^j H^T (S_k^j)^{-1}, \quad (34)$$

with S_k^j being the covariance of the j th innovation process

$$S_k^j = H P_{k|k-1}^j H^T + R. \quad (35)$$

The updated error covariance matrix is calculated using

$$P_{k|k}^j = P_{k|k-1}^j - K_k^j H P_{k|k-1}^j \quad (36)$$

If the measurement z_k is not available (e.g., due to sensor error, blind range of the sensors, etc.), equations (32)-(36) are omitted and only time propagation (27)-(30) is performed.

4) *Mode Probability Update:* The prior probability that u_M^j is correct (the system is in regime j) is

$$\text{Prob}\{u_M^j | \mathcal{Z}_0\} = \mu_0^j, \quad j = 1, \dots, p \quad (37)$$

where \mathcal{Z}_0 is some a priori known information and $\sum_{j=1}^p \mu_0^j = 1$, since the correct law is $u_M \in \{u_M^1, \dots, u_M^p\}$.

Using Bayes' rule, given the measurement data $z_{1:k} \triangleq \{z_i; i = 1, \dots, k\}$ up to time k , the posterior probability μ_k^j of the mode j being correct is given by [6]

$$\mu_k^j = p(z_k | z_{1:k-1}, u_M^j) \mu_{k-1}^j / p(z_k | z_{1:k-1}). \quad (38)$$

Using (37) and applying the total probability, (38) results in

$$\mu_k^j = \Lambda_k^j \mu_{k-1}^j / \sum_{i=1}^{n_r} \Lambda_k^i \mu_{k-1}^i, \quad j = 1, \dots, p, \quad (39)$$

where $\Lambda_k^j \triangleq p(z_k | z_{1:k-1}, u_M^j)$ is the j th regime-conditioned likelihood function, given by

$$\Lambda_k^j = p(\nu_k^j) = \mathcal{N}(\nu_k^j; \mathbf{0}, S_k^j), \quad (40)$$

where ν_k^j and S_k^j are the innovation and its covariance from the j th mode-matched filter. It is obvious that $\mu_k^j \geq 0, \forall j \in \{1, \dots, p\}$ and $\sum_{i=1}^p \mu_k^i = 1, \forall k \geq 0$. Note that in a nonlinear and/or non-Gaussian setting, Gaussian likelihood functions are used, although they are clearly approximations [6].

B. MMAC Configuration for Target Evasion

In the MMAC approach [8], the estimation of each elementary filter is fed into a "controller" (optimal evasion strategy in our case) and the total control command can be determined by one of the following approaches:

a) MMSE - minimum mean square error, where the target

control command u_T is a weighted average of controls from each filter-matched controller in the bank, i.e.,

$$u_T^{MMSE} = \sum_{j=1}^p \mu_k^j u_T^{*j}, \quad (41)$$

where $u_T^{*j} = a_T^{max} \text{sign}(s_{MT}^j) \text{sign}(Z_{MT}^j)$, s_{MT}^j and Z_{MT}^j represent, respectively, the switching function (13) and the ZEM distance (14), both computed using the j th regime mode-matched estimate $\hat{x}_{k|k}^j$ at time step k .

b) MAP - maximum a posteriori probability. In the MAP sense, u_T is determined as the control associated with the maximum a posteriori probability, i.e.,

$$u_T^{MAP} = u_T^{*i}, \quad i = \text{argmax}_j(\mu_k^j). \quad (42)$$

Using this scheme, as the identification process of the missile's unknown parameters improves, the target's evasion will adapt to fit the identified missiles guidance strategy.

C. Implementation Issues

If collision course conditions hold, the range cannot be estimated from bearing-only measurements and thus the proposed MMAES cannot be properly implemented as an accurate estimate of t_{go} is required. For practical implementation, the time-to-go t_{go} is commonly approximated as

$$\tilde{t}_{go} \approx -r/V_r, \quad V_r < 0. \quad (43)$$

A poor range estimation might result in inappropriate timing of the optimal switches dictated by s_{MT} and thus in a poor evasion performance. One way to improve the accuracy of \tilde{t}_{go} is to improve range observability. Maneuvering away from the collision triangle, i.e., forcing the collision triangle to rotate, can improve the performance of the estimation process because, by altering the LOS, the bearing measurement will return some insights on the relative range.

It is obvious that $Z_{MT}(t)$ might cause chattering of u_T^* . This chattering might happen when $Z_{MT}(0) \simeq 0$ and uncertain states are used to compute Z_{MT} . Such unwanted chattering of u_T^* might lead to a non-rotating collision geometry. A workaround is to apply a dead-zone-like function on the sign of $Z_{MT}(t)$. By this, if $|Z_{MT}(t)| < \epsilon$, i.e., the ZEM distance is smaller than some prescribed value ϵ , the target will not change the direction of its maneuver command, unless the sign of $s_{MT}(t_{go})$ is changed or $|Z_{MT}(t)| \geq \epsilon$.

Note that in the linear setting, $|Z_{MT}(t)|$ is a monotonically increasing function of time t , see (17). By letting $\epsilon \rightarrow \infty$, we actually assume that the monotonically increasing property also holds for the nonlinear engagement. In that case, the optimal nonlinear evasion strategy (19) reduces to

$$u_T^* \approx a_T^{max} \text{sign}(s_{MT}) \text{sign}(Z_{MT}(0)), \quad Z_{MT}(0) \neq 0. \quad (44)$$

If $Z_{MT}(0) = 0$, $u_T^*(0)$ can be chosen arbitrarily as either u_T^{max} or $-u_T^{max}$, and $\text{sign}(Z_{MT}(0)) \triangleq \text{sign}(u_T^*(0))$ should be assumed constant in (44) throughout the engagement.

Let us now consider the above discussions within the MMAC setting. Each "controller" in the bank has its own switching function s_{MT}^j entirely defined by Φ_E^j . Each s_{MT}^j is evaluated using the t_{go} approximation, calculated for t_k

using (43) with $\hat{\mathbf{x}}_{k|k}^j$. If $Z_{MT}^j(0) \approx 0$ for all $j \in \{1, \dots, p\}$, then u_T^{*j} should be initialized with the same value ($\pm u_T^{max}$) for all j . By this, u_T , computed using MMSE or MAP, will not be affected by the initial transitions (convergence) of the probabilities μ_k^j , until the first switch occurs in any s_{MT}^j .

V. PERFORMANCE ANALYSIS

The performance of the proposed MMAES is evaluated here using numerical simulations and nonlinear kinematics.

A. Simulation Environment and Scenario

All engagements are initiated at a horizontal separation of 5000 m between the missile and the target, thus $r_0 = 5000$ m and $\lambda_0 = 0$ rad. Both missile and target have constant speed. The target's speed is $V_T = 300$ m/s and the missile's speed is $V_M = 600$ m/s. For the analysis we assume that the missile and the target have a first-order strictly proper dynamics with time constants $\tau_M = 0.2$ s and $\tau_T = 0.5$ s. In this case, matrices (4) degenerate to $\mathbf{A}_i = -1/\tau_i$, $\mathbf{B}_i = 1/\tau_i$, $\mathbf{C}_i = 1$ and $d_i = 0$, $\forall i \in \{M, T\}$. The target's maneuver capability is limited to 10 g (i.e., $u_T^{max} = 10$ g). No saturation is applied on the missile acceleration command u_M . Given the target's initial flight path angle γ_{T0} , the missile's initial flight path angle γ_{M0} is selected such that the initial collision conditions hold, i.e., $V_M \sin(\gamma_{M0} - \lambda_{M0}) - V_T \sin(\gamma_{T0} + \lambda_{T0}) = 0$.

We consider that the missile is guided to the target using perfect information and one of the following "classical" guidance laws: PN [9], APN [10], and OGL [11] with navigation gain $N_i' \in \{3, 3.5, 4, 4.5, 5\}$ for $i = \{\text{PN}, \text{APN}\}$ and weight $\alpha \in \{0.002, 0.004, 0.006, 0.008, 0.01\}$ for OGL¹. The MMAC regimes are matched exactly to the above guidance laws and parameters, requiring in total 15 EKFs to be run in parallel (i.e., $p = 15$). The prior probability of each regime is $\mu_0^j = 1/15$, $j = 1, \dots, p$.

The time propagation of the EKFs is performed at a frequency of 200Hz (i.e., $T_s = 1/200$ s). Measurements are acquired at a sampling frequency of 50 Hz (i.e., $T_s^m = 1/50$ s). The simulated measurement noises are with $\sigma_r = 10$ m and $\sigma_\lambda = 1$ mrad. The EKF tune parameters ψ^j , $j = 1, \dots, p$ have been chosen by numerical simulations. The integral in (31) is computed numerically using one-step forward integration, whereas the nonlinear EOM (27) are propagated using the RK4 algorithm. We assume a blind range of 50 m. If $r < 50$, no new measurements are acquired.

B. Sample Run

A sample run planar trajectories of a target employing MMAC with MMSE blending and bearing-only measurements evading from an APN-guided missile with $N'_{APN} = 4$ is presented in Fig. 2. The initial flight path angle of the target is $\gamma_{T0} = \pi/12$ rad. The resulting miss distance in this sample case is approximately 0.38 m. It can be seen that the applied evasion strategy forces the collision triangle to rotate with time, thus allowing to gain some insight on the range.

¹The weight α represents the ratio between the control effort and the miss distance in the quadratic cost function used in the OGL derivation.

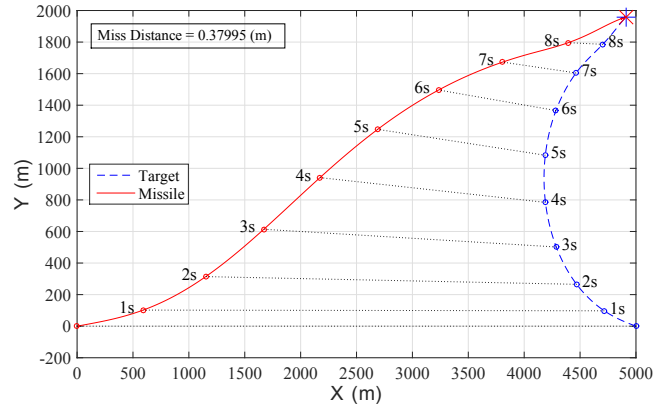


Fig. 2. Sample run planar engagement trajectories.

Figure 3 compares the timing of the optimal target maneuver switches with the switches obtained using the proposed MMAES. In this figure, the acceleration a_T (red line) is driven by the target's acceleration command u_T (black dashed line). The upper frame shows the optimal switches obtained when the target assumes perfect information, i.e., uses true states and is exactly matched to the active missile guidance strategy. The other four frames below represent the MMAES implemented with different MMAC/measurement combinations. It can be seen that while the MMAC with two measurements (i.e., r and λ) exhibits almost identically to the perfect information case, the MMAC with bearing-only measurements (i.e., λ) executes the optimal switches with a slight time delay of about 0.3 s. This is due to the poorer range estimation, thus poorer time-to-go approximation.

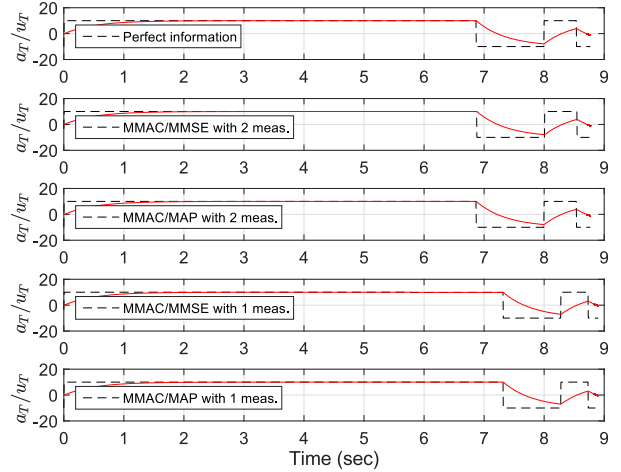


Fig. 3. Target acceleration profiles for different scenarios.

Figures 4-5 present the posterior probabilities of each guidance law being true and the probability of each guidance law parameter being correct as a function of time. In the two measurements case (single measurement case), the APN guidance law has been identified as the missile's guidance law after approximately 1 s (1.5 s) and the navigation gain has also been identified approximately around 2 s (5 s). Note that the obtained results for MMAC with MAP are very similar to those given in Fig. 2 and Figs. 4-5, thus omitted.

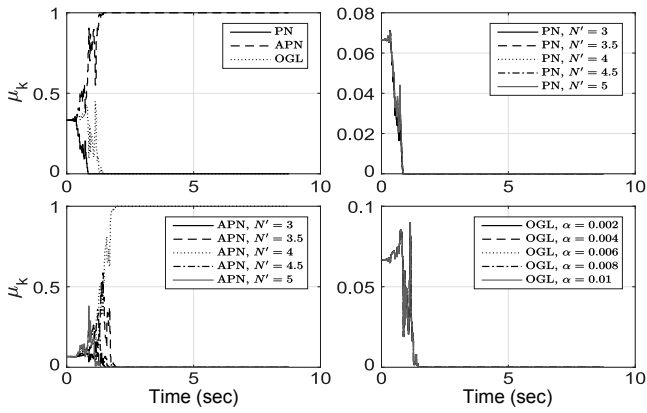


Fig. 4. Regime probabilities for two measurements.

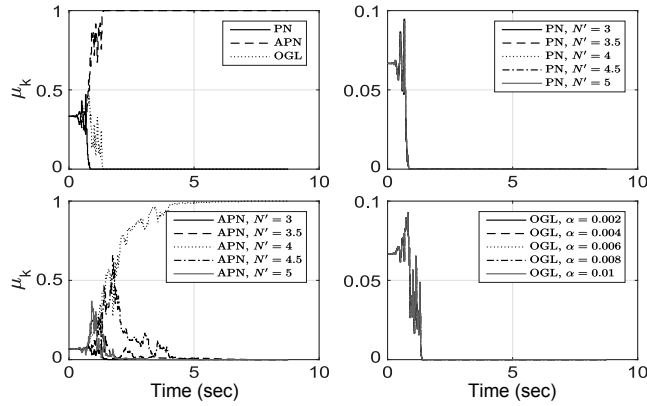


Fig. 5. Regime probabilities for bearings-only measurements.

C. Monte Carlo Study

The effect of the “estimator in the loop” on the evasion accuracy, for four different MMAC/measurement combinations, is evaluated here. The analysis is based on a set of 500 MC runs. The results are compared in terms of miss distances. The employed missile guidance law and its navigation gain is one of those presented in Sec. V-A. For each run, the active missile guidance was selected by using a uniform random number generator. The studied engagement is symmetric with respect to X-axis, thus only positive γ_{T0} are considered being drawn from a uniform distribution on $[0, \pi/6]$ rad. Each MC run differs from the other run by different γ_{T0} , by different noise seeds, and by different initial guesses, $\hat{\mathbf{x}}_{0|0}$, assigned to the estimators. These guesses are sampled from a Gaussian distribution, i.e., $\hat{\mathbf{x}}_{0|0} \sim \mathcal{N}(\mathbf{x}_0, \mathbf{P}_{0|0})$, where \mathbf{x}_0 is the true relative missile-target initial state and $\mathbf{P}_{0|0} = \text{diag}\{50^2, (3\pi/180)^2, (2.5g)^2, (3\pi/180)^2, 60^2\}$ is the initial estimation error covariance matrix of the filters.

Figure 6 shows the obtained miss distances by means of the cumulative distribution function (CDF). The expected linear miss, given by (18), is also considered for illustration. It can be seen that the cases with two measurements achieve almost identical evasion performances as the case when the target has perfect information and that The MMSE and MAP criterion for the MMAC achieve very similar evasion performances. It is also evident that the performance deteriorates when bearing-only measurements are considered.

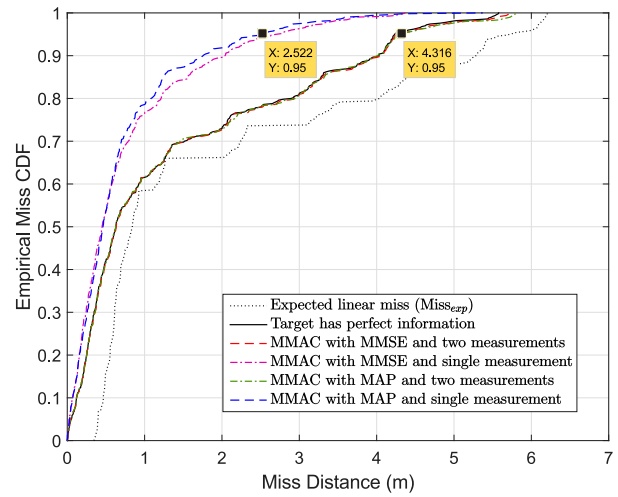


Fig. 6. Cumulative distribution function of the estimated miss distance.

VI. CONCLUSION

A multiple model adaptive evasion strategy for a target aircraft from a homing missile employing a linear guidance strategy has been presented. Considering different sets of noisy measurements, the evasion performance of the proposed approach has been numerically studied, through Monte Carlo simulations, for missile employing classical guidance laws such as PN, APN, and OGL, using nonlinear kinematics and missile provided perfect information. For the missile and target having first-order dynamics, it was shown that the degradation in avoidance performance from a homing missile may not be as serious as it could have been anticipated, even when the target has limited maneuver capability and carries sensors that provide bearing-only measurements.

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