Estimation Enhancement by Cooperatively Imposing Relative Intercept Angles

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Cooperative estimation/guidance for a team of missiles is the topic of this paper. An example scenario is considered, where an aircraft simultaneously launches several cooperative defending missiles as a countermeasure against an attacking homing missile. A new reduced-order estimation scheme based on information sharing to cooperatively estimate the relative states and the unknown parameters of the attacking missile is proposed. Each defending missile shares its own noise-corrupted line of sight angle measurement with the rest of the team. The observability of this multi-line-of-sight measuring environment is enhanced by cooperatively imposing nonnegative relative intercept angles between consecutive defenders. The ability of the proposed strategy to protect the targeted aircraft is studied for a two-defender case via extensive Monte Carlo simulations. The effect of different values of commanded relative intercept angle on the pure estimation as well as on the intertwined guidance-estimation performance is carefully analyzed for various considerations of defenders' maneuverability.

Nomenclature

a	=	normal acceleration
A, B, C, D	=	state space representation of the closed-loop dynamics
$\mathcal{D}_c$	=	set of considered relative intercept angle commands
f	=	nonlinear equations of motion
F	=	state transition Jacobian matrix
h	=	measurement function
H	=	measurement Jacobian matrix
J	=	cost function
k	=	instantaneous projection (linearization) coeficient
K	=	Kalman filter gain matrix
N'	=	navigation gain of the attacking missile
$N^{u_i}_{Z_j}$, $N^{u_i}_{\Delta Z_j}$	=	i-th defender's navigation gains associated with the j-th defender's zero-effort miss and zero effort angle error, respectively
$\mathcal{N}$	=	Gaussian distribution
n	=	total number of considered defenders
n_a	=	number of active defenders
P	=	state error covariance matrix
Q	=	covariance matrix of the equivalent discrete process noise
R	=	measurement covariance matrix
t, t^{go}, t^{f}	=	time, time-to-go, and interception time, respectively

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T = sampling period
V = speed
v = measurement noise
w = nonnegative weight
u = acceleration command
$\mathcal{U}$ = set of considered maneuverability limitations
x = state vector
x^a = state vector of the vehicle's dynamics
x_i, y_i = inertial coordinates of the i-th entity position
Δx_{ij}, Δy_{ij} = relative displacements of the i-th entity from the j-th entity
z = measurement
Z = zero-effort miss/flight-path angle
α = weight on the defender's miss
η = weight on the defender's control effort
β = weight on the defenders' relative intercept angle
ξ = defender-missile relative displacement normal to the initial LOS
γ = flight-path angle
$\gamma_{d_i m}$ = angle between the i-th defender and the missile
ς = boolean variable indicating the status of the defender
Δ_c = relative intercept angle requirement
θ = normalized time-to-go
δ_m = unknown missile guidance parameter(s)
ε = ratio between the weight on the missile's control effort and the miss
λ = angle between the line of sight and the X_I axis
ρ = range
σ_λ^2 = LOS angle measurement noise variance
τ = time constant
Φ = transition matrix
ψ = time-varying function
Ψ = tuning parameter of the Kalman filter
$[0]$ = matrix of zeros with indicated dimension
$\hat{(\cdot)}$ = estimated value
$\bar{(\cdot)}$ = approximated value

Subscripts

d_i = i-th defending missile (defender)
m = attacking missile (missile)
t = target aircraft (target)
k = step of the discrete time t_k
ρ, λ = along and normal to the line of sight

Superscripts

err = error
max = maximal value
$\star$ = perfect information assumption
$\dagger$ = indirect measurement model
I = inertial coordinate frame
R = relative (polar) coordinate frame

I. Introduction

Modern highly sophisticated aerial threats such as anti-aircraft missile, tactical ballistic missile, and unmanned aerial vehicle (UAV) are able to engage and destroy a large class of urban and aerial targets [1]. Such threats are usually characterized by low observability and high maneuverability. Advanced air-defense missile systems include radar-guided surface-to-air missiles and modern fighter aircraft armed with various sensor-guided defending missiles. The major requirement for such defense systems, designed to negate these threats, is improved interception performance, i.e., attaining small miss distance to ensure destruction of the adversary. However, the performance of any guidance system, aimed to achieve small miss distances against maneuvering threats, is highly reliant on the knowledge of the opponent's acceleration, relative state, and other uncertain parameters related to the opponent. For instance, the opponent's acceleration cannot be directly measured and therefore has to be estimated based on available noise-corrupted measurements. The unavoidable estimation errors may degrade the interception performance. Development of advanced sensor systems, use of more agile missiles, and/or deployment of more lethal warhead are some of the few possible options to deal with the problem of uncertainty. However, these options might be very often too complex, heavy, and expensive. An alternative is to design more sophisticated guidance and estimation algorithms to improve the guidance system of inexpensive missiles, without deteriorating the required interception accuracy.

Gimballed electro-optical seekers are usually positioned at the front tip of the missile. Often, the size of the seeker and its supporting systems dictate the missile's front tip shape, which in turn affects maneuverability, volume, and aerodynamic constraints. Most tactical missiles are equipped with affordable infra-red (IR) sensors, which allow to measure the line-of-sight (LOS) angle between the pursuer and the opponent. Fixed opponent (target) localization using bearings-only measurements is an observable process even without an observer (e.g., missile) maneuver. Oshman and Davidson [2] used the determinant of the Fisher information matrix to optimize the observer's trajectories for bearings-only stationary opponent localization problem. However, the estimation performance for a target tracking problem, in the presence of maneuver uncertainty and noise-corrupted bearings-only measurements, is limited [3–5]. Nardone and Aidala [3] and Hepner and Geering [4] showed that certain types of maneuvers do not necessarily guarantee observability in target tracking problems where only a single LOS angle measurement is available. For example, employing proportional navigation (PN) guidance law attempts to null the LOS rate. As a consequence, range and range-rate are not observable [5]. A solution to improve range observability is to maneuver away from the collision course. This causes the LOS to rotate which in turn gives some insights on the relative range. Based on this idea, Battistini and Shima [6] proposed a guidance logic which exploits the information content of the error covariance matrix's eigenvalues.

In recent years, multi-missile counterattack against aerial threats has been conceived as a very effective way to survive, as it may significantly increase the success rate of such countermeasure [7]. In scenarios where multiple missiles can share their respective LOS angle measurements, the estimation performance can be improved by exploiting the triangulation structure [8–12]. The estimation quality, however, strongly depends upon the missiles' trajectories and hence on the implemented guidance law. Shaferman and Oshman [11] proposed two estimation methods for cooperative target tracking based on information sharing of multiple missiles. It was assumed that the missiles are guided towards the target via a given one-on-one guidance law, and that only the estimation is performed cooperatively. However, guidance and estimation are mutually intertwined. Neglecting the effect of the one onto the other and vice versa may have severe consequences. For example, when all missiles employ the same one-on-one guidance law (as considered in [11]) and are all fired with the same initial conditions, then the resulting missile trajectories coincide (up to some unmodeled disturbances). As a consequence, all sensors will measure the same quantity, causing the triangulation technique to fail and thus making the range unobservable. This, in turn, might result in poor interception performance. The work of [11] was recently extended in [12] by the same authors, where the above issue was resolved by introducing the concept of staggered launch of the missiles. In this work, the optimal staggering was derived based on a linear model and a deterministic approximation of the stochastic estimation process. Chen and Xu [8, 9] analyzed the observability issue in a double-LOS relative navigation setup. They concluded that if the separation angle between the LOS vectors is too small, the relative navigation system may become weakly observable or even unobservable. This problem was addressed for the two missiles case by Liu et al. [10] by modulating the LOS angle through a performance index. The missile with large initial LOS angle maximizes this index while the other one minimizes it. By this, the separation angle of both LOS vectors during the engagement is increased and the estimation is improved.

The work presented in this paper sets the above concepts into a specific problem of aircraft protection from an attacking homing missile. In this paper, the aforementioned concepts of multimissile attack and information sharing are related to a specific problem of an active aircraft defense from an an incoming homing missile. The aircraft in this study may be a manned or an unmanned aerial vehicle. Among systems developed in the past decades to increase aircraft's protection capabilities are electronic countermeasures (jammers) and various kinds of decoys (e.g., chaff or flares). The aircraft may also perform evasive maneuvers, which can be either arbitrary [13–15] or optimally adjusted against the incoming missile [16–18]. In case of arbitrary maneuvers, Zarchan [13] suggested a random telegraph approach. A periodic sine wave maneuver with a random phase but a frequency that is matched to the missile's navigation gain and time constant were discussed by Zarchan [14] and Ohlmeyer [15]. Shima [16] derived optimal evasive maneuvers against a homing missile employing a known linear guidance law. This work was recently extended by Turetsky and Shima [17] for a case where the missile performs multiple switches between known linear guidance laws and by Fonod and Shima [18] for a case where the guidance law of the missile is unknown. All these countermeasures may not provide sufficient protection against agile and advanced adversarial systems. An alternative solution is to launch one or several defending missiles (defenders) to intercept the incoming threat before it reaches the aircraft. As the defenders and the targeted aircraft (target) can cooperate with each other to intercept the attacking missile, such an active self-defense system results in a special multiple-body pursuit-evasion guidance scenario which is different from the typical one-on-one engagements.

Most of the works in the literature on active aircraft protection deal with guidance strategies for the aircraft defender team as a three-body problem (target, missile, and a single defender). The common assumptions are that the attacking missile is homing onto the target using perfect information, is unaware of the defender(s), and employs a linear one-on-one guidance law. Asher and Matuszewski [19] discussed the possible application of an optimal control-based guidance law for such a scenario. Boyell [20] presented the first results on kinematics of the three-body problem, where closed form relations were derived for constant-bearing collision courses. Shneydor [21], in his comments on Boyell's work [20], simplified one of the collision conditions. In a later work under the same assumptions, Boyell [22] obtained a closed-form expression for the intercept point in target-centered coordinates. Shinar and Silberman [23] investigated the three-body problem as a three-player two-team game. Rusnak et al. [24] studied optimal strategies for the three-body problem (including the missile) in a linear quadratic (LQ) game setting. It was shown in [24] that as the weight on the defender command tends to zero, the optimal strategies of the target and the missile are identical in form to those obtained in the one-on-one game without the defender. Ratnoo and Shima [25] proposed an interception method where the defending missile is commanded to be on the LOS between the attacking missile and the aircraft for all time, while the aircraft follows some predetermined trajectory. This command to LOS (CLOS) defender guidance resulted in improved performance against missiles using proportional navigation guidance law. However, this approach requires the defender to have at least the same speed as the attacker. Yamasaki et al. [26] presented a modified CLOS guidance law which further improves the defender CLOS performance. Shima [16] derived an optimal control based cooperative evasion and pursuit strategy for an aircraft and its defending missile for the case where the attacking missile uses a known linear guidance strategy. The sole objective was to minimize the defender-missile miss distance. Prokopov and Shima [27] analyzed different types of cooperation, assuming the attacking missile is unaware of the presence of the defender and its guidance law is known. In contrast to [16], the methods proposed in [27] additionally minimize the target's acceleration requirements. Perelman et al. [28] obtained an analytical solution to the LQ differential game and studied the conditions for the existence of a saddle point solution. Garcia et al. [29] extended these results by allowing the defender's turning rate to be constrained.

All the previous cooperative guidance laws on the three-body problem assumed in their derivation that perfect information is available both for the homing missile and for the target-defender team. Shaferman and Shima [30] presented a multiple model adaptive guidance/estimation approach to identify the active guidance strategy of the incoming missile. In this work, the defender's guidance law is matched to the identified guidance strategy of the missile and the target's maneuver minimizes the control effort requirements of the defender by performing an appropriately timed single direction maneuver switch. Furthermore, the method presented in [30] assumes that the target and the defender are equipped with either radar or electro-optic sensors.

While all the works presented earlier emphasize the optimization and/or cooperation of the aircraft with a single defender, this paper considers a case when multiple defenders are fired simultaneously to cooperatively intercept the attacking missile. We assume that the defenders have limited maneuver capabilities and are equipped with affordable IR sensors only, which provide noisy LOS angle measurements. We also assume that each defender can share without any delay its own-ship measurement with the other defenders and that the target-defenders team has imperfect a priori information about the relative state and guidance parameters of the missile. The missile is assumed to be guided towards the target using one of the classical missile guidance laws of PN, augmented PN (APN), and optimal guidance law (OGL). However, unlike in [30], we consider a "fire and forget" policy which does not require the target to be equipped with any sensor system that measures the bearing and/or the range to the missile. The only assumptions made with respect to the target are that, prior to launching the defenders, its future maneuver strategy was successfully communicated to all defenders and that its location is known to the defenders to high accuracy. For sake of generality, we assume that the guidance strategy of the target is arbitrary.

In this paper, we significantly extend the cooperative estimation-enhancing guidance concept of [31] by re-deriving the solution for a team of $n > 1$ aircraft defending missiles and by demonstrating the viability of the proposed concept by additional simulation results. We propose a new reduced-order estimation scheme based on information sharing to cooperatively estimate all the defenders-missile relative states and missile's related parameters such as speed, acceleration, flight-path angle, and its guidance-related parameters. This scheme enables the entire estimation algorithm to be implemented in a centralized manner on a sole defender. Therefore, the computational requirements on the on-board computer of the remaining defenders can be significantly reduced. Considering the observability issues of multi LOS measuring environment discussed earlier, and motivated by the recent developments in terminal intercept angle missile guidance theory [32–34], the idea in this paper is to enhance/ensure observability by imposing nonnegative intercept angle constraints between the defenders. This can be achieved for each defender separately using one of the available one-on-one terminal intercept angle guidance laws, e.g., Ryoo et al. [32] and Shaferman and Shima [33]. Or, better yet and as done in this paper, one can achieve the same goal using the optimal guidance law of Shaferman and Shima [34] which can impose a predetermined relative intercept angle between consecutive defenders based on explicit cooperation of all defenders. By imposing nonnegative relative intercept angle constraints, the terminal trajectory separation between the defenders can be ensured and the observability improved. Additionally, by exploiting the knowledge of the known future target maneuver and the fact that the attacking missile is homing onto the target, we relax the known constant acceleration assumption in the derivation of the guidance law presented in [34]. This strategy also helps to minimize the defenders' control effort as the intercept points of the defenders are implicitly taken into account in the missile's acceleration profile prediction phase, which is performed by numerical integration of the relevant engagement's equations and the use of the most recent estimates. In this paper, we extend the analysis presented in [34], where perfect information and saturation-free scenario was considered. Through an intensive numerical study, we rigorously analyze the effect of different values of the commanded relative intercept angle on the pure estimation as well as on the intertwined guidance-estimation performance, while also evaluating different acceleration requirements for the defenders.

The remainder of this paper is organized as follows. The next section presents the mathematical models of the target-defenders-missile engagement. The joint estimator is derived in Sec. III, followed by the presentation of the cooperative guidance law for the defenders in Sec. IV. A comprehensive performance analysis is done in Sec. V, followed by concluding remarks.

II. Multiple-Body Engagement Description

The considered scenario consists of several entities (also referred to as bodies or vehicles): an attacking missile, an evading target aircraft, and a group of $n > 1$ aircraft defending missiles. For brevity, the target aircraft is referred to as target, the attacking missile as missile, and the defending missiles as defenders. The target and the n defenders form a team against the missile.

Next, the full kinematics and dynamics equations of the target-defenders-missile engagement are presented. Then, we introduce the timeline and the considered physical measurement model.

II.A. Kinematics and Dynamics

We consider skid-to-turn and roll-stabilized vehicles. The motion of the target, all the n defenders, and the missile is assumed to transpire in the same plane. In Figure 1 a schematic view of the considered planar point mass engagement geometry is shown, where $X_I{-}O_I{-}Y_I$ represents a Cartesian inertial reference frame. The missile, i-th defender, and target related variables are denoted by the subscripts m, d_i, and t, respectively. The speed, normal acceleration, and flight-path angles are denoted by V, a, and γ, respectively. The range between the target and the missile and between the i-th defender and the missile is denoted as ρ_{tm} and ρ_{d_im}, respectively. The angle between the target's line-of-sight (LOS) to the missile and the X_I axis is denoted as λ_{tm}, while that between the i-th defender's LOS to the missile and the X_I axis is denoted as λ_{d_im}.

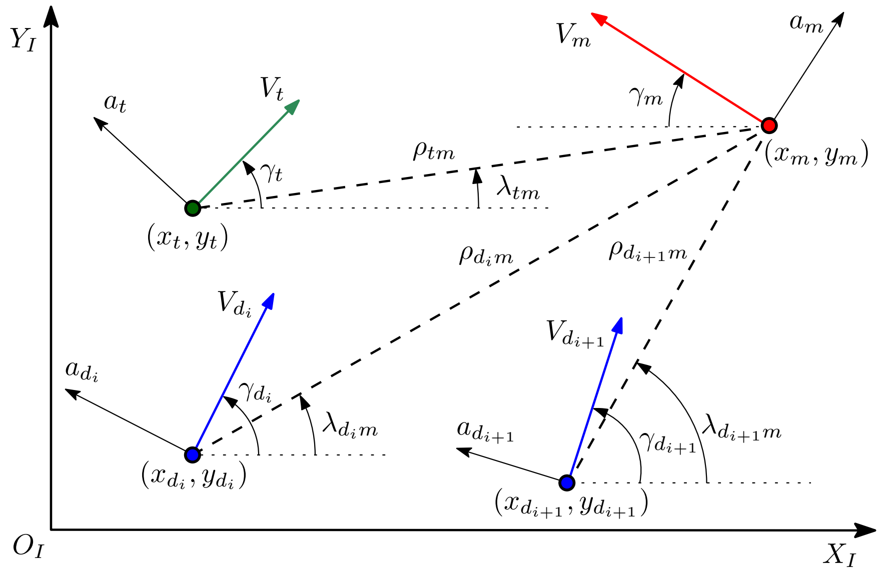


Figure 1: Planar target-defenders-missile engagement geometry.

The missile and the defenders are considered to be from a similar class of vehicles, with their speeds higher than that of the target, i.e., $V_i > V_t$, $\forall i \in \{m, d_1, \ldots, d_n\}$.

We assume that the target's and defenders' own inertial state vector

$$x_i^I = \begin{bmatrix} x_i & y_i & a_i & \gamma_i \end{bmatrix}^T, \qquad i \in \{t, d_1, \ldots, d_n\}, \tag{1}$$

is known to a very high accuracy (e.g., using inertial navigation system and/or GPS sensors), and that the target and each defender can transmit its own inertial state vector to other defenders in the team without any delay. We also assume that the target's speed is known or is transmitted to the defenders.

Remark 1. Only the relative positions between the cooperative vehicles (target and defenders) will be important in the rest of the derivation. As long as all vehicles in the team can track each other accurately, the above assumption about x_i^I is not invalidated.

Neglecting the gravitational force, the engagement kinematics, expressed in a polar coordinate system $(\rho_{im}, \lambda_{im})$ attached to the i-th entity, is:

$$\begin{cases} \dot{\rho}_{im} = V_{\rho im} \\ \dot{\lambda}_{im} = V_{\lambda im}/\rho_{im} \end{cases}, \qquad i \in \{t, d_1, \ldots, d_n\}, \tag{2}$$

where the respective relative velocities along and perpendicular to the LOS are

$$V_{\rho im} = -V_i \cos(\gamma_i - \lambda_{im}) - V_m \cos(\gamma_m + \lambda_{im}), \tag{3a}$$
$$V_{\lambda im} = -V_i \sin(\gamma_i - \lambda_{im}) + V_m \sin(\gamma_m + \lambda_{im}). \tag{3b}$$

During the endgame, all the vehicles are assumed to move at a constant speed and to perform lateral maneuvers only. Arbitrary-order linear dynamics is assumed for all vehicles

$$\begin{cases} \dot{x}_i^a = A_i x_i^a + B_i u_i \\ a_i = C_i x_i^a + D_i u_i \\ \dot{\gamma}_i = a_i/V_i \end{cases}, \qquad i \in \{t, m, d_1, \ldots, d_n\}, \tag{4}$$

where $x_i^a \in \mathbb{R}^{n_i}$ is the internal state vector of the i-th vehicle's dynamics, a_i and u_i are the i-th entity's normal acceleration and acceleration command, respectively. The term $C_i x_i^a$ is denoted as a_{is} and represents, if it exists, the part of the acceleration with dynamics (for example, an angle of attack generating lift). The second part of the acceleration, i.e. $D_i u_i$, represents the direct lift, which can be obtained immediately from deflection of the steering mechanism such as the canard or tail (neglecting servo dynamics).

We assume that the target's and defenders' maneuverabilities are limited to

$$|u_i| \le u_i^{max}, \qquad i \in \{t, d_1, \ldots, d_n\}, \tag{5}$$

where $u_i^{max} > 0$ is the i-th vehicle's maximal acceleration. No saturation is considered for u_m.

II.B. Timeline and Time-to-go

The running time is denoted as t. The endgame initiates at $t = 0$ with $\dot{\rho}_{im}(t = 0) < 0$, $\forall i \in \{t, d_1, \ldots, d_n\}$. The particular engagement terminates at $t = t_{im}^f$, where t_{im}^f is the target-missile or defender-missile interception time, formally defined as

$$t_{im}^f = \arg\inf_{t>0}\{\rho_{im}(t)V_{\rho im}(t) = 0\}, \qquad i \in \{t, d_1, \ldots, d_n\}. \tag{6}$$

The interception time t_{im}^f allows to define the true nonnegative time-to-go as

$$t_{im}^{go} = \begin{cases} t_{im}^f - t, & t \le t_{im}^f \\ 0, & t > t_{im}^f \end{cases}, \qquad i \in \{t, d_1, \ldots, d_n\}. \tag{7}$$

At $t = t_{im}^f$, the separation $\rho_{im}(t_{im}^f)$ is minimal and is referred to as "miss distance" or compactly as "miss".

Without loss of generality, we assume that the target-missile engagement terminates after that of the defenders-missile, i.e., $t_{d_i m}^f < t_{tm}^f$, $\forall i \in \{1, \ldots, n\}$, and that the defenders are numbered based on their interception times, satisfying

$$t_{d_1 m}^f \le t_{d_2 m}^f \le \ldots \le t_{d_n m}^f. \tag{8}$$

II.C. Physical Measurement Model

The bearing measurement is a pre-dominant one in missile guidance applications and it requires a relatively inexpensive seeker. Therefore, we assume that each defender is only equipped with an IR sensor that measures the LOS angle, i.e., the i-th defender measures only $\lambda_{d_i m}$. The measurements are assumed to be contaminated by a zero-mean white Gaussian noise with standard deviation $\sigma_{\lambda d_i m}$ and all being acquired at the same discrete-time $t = t_k \triangleq k \cdot T$, where $T > 0$ is the measurement sampling period.

Based on the above assumptions, the physical measurement equation of the i-th defender is

$$z_{d_i;k} = h_{d_i}(x_k) + v_{d_i;k} = \lambda_{d_i m;k} + v_{d_i;k}, \qquad i \in \{1, \ldots, n\}, \tag{9}$$

where the state vector x_k, used for estimation, will be defined later, and

$$v_{d_i;k} \sim \mathcal{N}(0, \sigma_{\lambda d_i m}^2).$$

In Eq. (9), and in the rest of the paper, the discrete time step is indicated (if unavoidable) by a subscript k, separated by a semicolon. The noise sequences $v_{d_i;k}$, $i \in 1, \ldots, n$, are assumed to be mutually independent.

We assume that each defender can transmit its own-ship measurement to the rest of the team without any delay. Additionally, we assume that from the time when the i-th defender passes the missile, i.e., $t_{d_i m}^{go} = 0$, this defender does not transmit its measurement nor its relative position to the rest of the team. The status of the i-th measurement is characterized by the boolean variable ς_i defined as

$$\varsigma_i = \begin{cases} 1 & \text{if measurement available,} \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

The advantage and utilization of the above measurements sharing concept will be discussed in more details in the next section.

III. Reduced-order Estimator Design

In practical interceptions with noisy measurements and incomplete information about the opponent, an estimator becomes an inevitable part of any advanced guidance system. The homing accuracy of such guidance system is then restricted by the performance of the estimator. Based on the separation theorem [35] and the associated certainty equivalence principle [36], the common practice is to design the guidance laws and the estimators separately.

The estimator can be implemented using a variety of techniques, e.g., various variants of the (extended) Kalman filter [37], divided difference filter [38], sliding mode observer/differentiator [39], particle filter [11, 12], etc. If the measurements can be shared between the defenders, then the estimation can be performed cooperatively and in a decentralized manner as done in [10–12]. In these references, it was assumed that each teammate has its own estimator and the computed state estimates are shared within the team. In this section, we propose a single, reduced-order, estimator design method to jointly estimate the defenders' state vector using shared bearings-only measurements.

III.A. Assumptions on the Missile Guidance

We assume that prior to firing the defending missiles, the target has acquired some intelligence about the missile's active guidance law and that this information is passed to the defenders. On the other hand, we assume that the corresponding parameters of this guidance law are unknown, and thus need to be estimated.

For simplicity of exposition, we will focus on the three most representative missile guidance laws of PN [40], APN [41], and OGL [42]. However, the derivation can be extended to other missile guidance laws using the same formulation and similar derivation steps.

The guidance laws of PN, APN, and OGL have the following form

$$u_m = N'_j \frac{Z_j}{(t_{tm}^{go})^2 \cos(\gamma_m + \lambda_{tm})}, \quad j \in \{\mathrm{PN}, \mathrm{APN}, \mathrm{OGL}\}, \tag{11}$$

where N' is the effective navigation gain, Z is the missile's zero-effort-miss (ZEM) distance, and t_{tm}^{go} is given by

$$t_{tm}^{go} = -\rho_{tm}/V_{\rho tm}, \quad V_{\rho tm} < 0. \tag{12}$$

The expression for the ZEM distance is different for each guidance law, i.e.,

$$Z_{PN} = V_{\rho tm}\dot{\lambda}_{tm}(t_{tm}^{go})^2, \tag{13a}$$

$$Z_{APN} = Z_{PN} + (t_{tm}^{go})^2 a_t \cos(\gamma_t - \lambda_{tm})/2, \tag{13b}$$

$$Z_{OGL} = Z_{APN} - \tau_m^2 \psi(\theta_{tm}) a_{ms} \cos(\gamma_m + \lambda_{tm}), \tag{13c}$$

where $\psi(\theta_{tm})$ is an exponential-like function of the normalized target-missile time-to-go θ_{tm}, given by

$$\psi(\theta_{tm}) = e^{-\theta_{tm}} + \theta_{tm} - 1, \qquad \theta_{tm} = t_{tm}^{go}/\tau_m. \tag{14}$$

and $\dot{\lambda}_{tm}$ and $V_{\rho tm}$ are given by Eqs. (2) and (3a), respectively.

The navigation gains N'_{PN} and N'_{APN} are constant, whereas N'_{OGL} is defined as a function of θ_{tm} as follows

$$N'_{OGL} = \frac{6\theta_{tm}^2 \psi(\theta_{tm})}{3 - 6\theta_{tm}\left(\psi(\theta_{tm}) + e^{-\theta_{tm}}\right) + 2\theta_{tm}^3 - 3e^{-2\theta_{tm}} + 6\varepsilon\tau_m^{-3}}, \tag{15}$$

where ε controls the ratio between the weights on the missile's control effort and miss in the LQ cost used in the OGL derivation [43].

We assume that the missile is not aware about the presence of the defenders, it is not trying to evade from them, and is guided towards the target via one of the classical guidance laws of PN, APN, or OGL with fixed guidance parameter N'_{PN}, N'_{APN}, or ε. These assumptions are realistic as most missiles can only track a sole target and are designed to intercept the tracked target [30].

Remark 2. If intelligence about the incoming missile is missing or is only partial, then one can resort to online identification techniques. For instance, a multiple-model adaptive estimation (MMAE) based approach can be used to identify the missile's fixed[a] guidance strategy [18, 30], or, in cases when the missile switches between different guidance strategies, an interactive multiple-model (IMM) based approach can be considered [11, 12, 44]. Both of these approaches require the target to be equipped with sensors that can measure bearing and/or range to the missile. Consequently, if these measurements can be shared with the defenders without any delay, then they can be used to improve the estimation accuracy of the defenders, see for instance [30] for discussion about advantages of such target-defender sharing concept.

III.B. Estimation Model

The i-th defender's state vector of the missile in polar coordinates is

$$x^R_{d_im} = \begin{bmatrix}\rho_{d_im} & \lambda_{d_im} & \gamma_m & x^a_m & V_m & \delta_m\end{bmatrix}^T, \tag{16}$$

where δ_m represents the unknown guidance parameter of the missile. In our case, δ_m may stand for N'_{PN}, N'_{APN}, or ε, depending on the identified guidance law.

In this paper, instead of designing n estimators, i.e., for each $x^R_{d_im}$ a separate one, we design a single estimator for the joint defenders' state, defined as

$$x^R_{dm} = \begin{bmatrix}\rho_{d_1m} \dots \rho_{d_nm} & \lambda_{d_1m} \dots \lambda_{d_nm} & x^a_m & \gamma_m & V_m & \delta_m\end{bmatrix}^T. \tag{17}$$

It is obvious that $\dim(x^R_{dm}) < \dim(x^R_{d_1m}) + \ldots + \dim(x^R_{d_nm})$. Thus, estimating x^R_{dm} instead of $x^R_{d_im}$, $i = 1, \ldots, n$ will require less computational effort because the parameters directly related to the missile (i.e., x^a_m, γ_m, V_m and δ_m) are not redundantly estimated by each defender in the team. In the rest of the paper, to avoid excessive indexing, we will denote x^R_{dm} as x.

Based on the constant missile speed and missile guidance parameters assumption, the model used for estimation is given by the following set of equations

$$\begin{cases} \dot\rho_{d_1m} = V_{\rho d_1m} \\ \qquad\vdots \\ \dot\rho_{d_nm} = V_{\rho d_nm} \\ \dot\lambda_{d_1m} = V_{\lambda d_1m}/\rho_{d_1m} \\ \qquad\vdots \\ \dot\lambda_{d_nm} = V_{\lambda d_nm}/\rho_{d_nm} \\ \dot x^a_m = A_m x^a_m + B_m u_m(x^R_{tm}, \delta_m) \\ \dot\gamma_m = \left(C_m x^a_m + D_m u_m(x^R_{tm}, \delta_m)\right)/V_m \\ \dot V_m = 0 \\ \dot\delta_m = 0 \end{cases}, \tag{18}$$

where $V_{\rho d_im}$ and $V_{\lambda d_im}$ are given in Eqs. (3a) and (3b), respectively, and $u_m(x^R_{tm}, \delta_m)$ is the missile's acceleration command given by (11). This command is a function of the target-missile relative state x^R_{tm} and the guidance parameter δ_m. The missile's relative state vector of the target in polar coordinates is

$$x^R_{tm} = \begin{bmatrix}\rho_{tm} & \lambda_{tm} & a_t & \gamma_t\end{bmatrix}^T. \tag{19}$$

For simplicity, we assume that the missile has perfect information about the target, but not vice versa. To use x^R_{tm} in Eq. (18), we need to compute x^R_{tm} using information that is available. As $x^I_i,\ i \in \{t, d_1, \ldots, d_n\}$ are assumed to be known accurately, thus, based on the triangulation technique, ρ_{tm} and λ_{tm} can be expressed using the most recent estimates of ρ_{d_im} and λ_{d_im} and the known relative positions as follows

$$\rho^{(i)}_{tm} = \sqrt{(\Delta X^i_{tm})^2 + (\Delta Y^i_{tm})^2}, \quad \lambda^{(i)}_{tm} = \operatorname{atan2}\left(\Delta Y^i_{tm}, \Delta X^i_{tm}\right), \tag{20}$$

[a]The missile is not switching between different guidance laws throughout the engagement.

where ΔX^i_{tm} is the horizontal and ΔY^i_{tm} is the vertical separation between the target and the missile from the i-th defender's perspective, respectively, given by

$$\Delta X^i_{tm} = \Delta x_{td_i} + \rho_{d_i m}\cos(\lambda_{d_i m}), \qquad \Delta x_{td_i} \triangleq x_{d_i} - x_t, \tag{21a}$$

$$\Delta Y^i_{tm} = \Delta y_{td_i} + \rho_{d_i m}\sin(\lambda_{d_i m}), \qquad \Delta y_{td_i} \triangleq y_{d_i} - y_t. \tag{21b}$$

It is obvious that any i-th perspective pair of $\rho^{(i)}_{tm}$ and $\lambda^{(i)}_{tm}$ can be used to compute ρ_{tm} and λ_{tm}. However, in order to increase the robustness of the algorithm, we propose the following weighted computation

$$\rho_{tm} = \sum_{i=1}^{n} w_i \rho^{(i)}_{tm}, \qquad \lambda_{tm} = \sum_{i=1}^{n} w_i \lambda^{(i)}_{tm}, \tag{22}$$

where w_i, $i = 1, 2, \ldots, n$ are nonnegative weights satisfying $\sum_{i=1}^{n} w_i = 1$. These weights shall be defined based on the designer needs. For example, when all the n sensors are deemed to be fault-free and equally accurate, then one can define $w_i = \varsigma_i / n_a$, where $n_a = \sum_{i=1}^{n} \varsigma_i$ is the number of active defenders. By doing so, Eq. (22) yields to a simple arithmetic average from the n_a active perspectives. Another way of defining the weights in Eq. (22) is to consider the actual estimation accuracy of $\rho_{d_i m}$ and/or $\lambda_{d_i m}$, e.g., by defining the weights as: $w_i = \varsigma_i \sigma_i^{-1} / \sum_{j=1}^{n}(\varsigma_j \sigma_j^{-1})$, where σ_i is the standard deviation of the estimation error of $\rho_{d_i m}$ or $\lambda_{d_i m}$, obtained from the filter. Finally, as a_t and γ_t are assumed to be known, the vector x^R_{tm} is fully defined by the available data.

Let us denote the vector that contains all the target-defenders relative positions at time t_k as

$$x^R_{td;k} = \begin{bmatrix} \Delta x_{td_1} & \ldots & \Delta x_{td_n} & \Delta y_{td_1} & \ldots & \Delta y_{td_n} \end{bmatrix}^T. \tag{23}$$

Using this notation, the discrete-time version of Eq. (18), used for the estimator design, can be compactly rewritten as

$$x_k = f_{k-1}(x_{k-1}, x^R_{td;k}), \tag{24}$$

where x_k is the defenders' joint state vector x^R_{dm} at time t_k, and f_{k-1} is a vector function derived by integrating of Eq. (18) from t_{k-1} to t_k.

Remark 3. In Eq. (18), we assumed that the parameters of the missile dynamics (A_m ,B_m, C_m, and D_m) are known. If this is not the case, the missile dynamics can be approximated by a first-order dynamics, i.e., $A_m = -1/\tau_m$, $B_m = 1/\tau_m$, $C_m = 1$, and $D_m = 0$, where the parameter τ_m can be treated similarly as V_m or δ_m, i.e., by adding τ_m as an additional constant state in Eq. (18).

III.C. Combined Measurement Model

If the defenders' measurements and their relative positions are shared, the defenders form a measuring baseline relative to the missile in space (see Fig. 1). Such information sharing might have very important benefits, such as improved observability due to different look angels at the missile and more measurements that are used to improve the estimation performance.

By exploiting the triangulation technique from the measurements perspective, we can express the model of the i-th physical measurement, given in Eq. (9), as a function of the j-th ($j \neq i$) defender's relative state and the known relative position between these two defenders, i.e., for $i, j \in \{1, \ldots, n\}$, $i \neq j$, we have

$$z_{d_i;k} = h^{\dagger}_{d_j}(x_k, x^R_{ji;k}) + v_{d_i} = \text{atan2}\left(\Delta Y_{ji}, \Delta X_{ji}\right) + v_{d_i}, \tag{25}$$

where

$$\Delta X_{ji} = \Delta x_{d_j d_i} + \rho_{d_j m}\cos(\lambda_{d_j m}), \qquad \Delta x_{d_j d_i} \triangleq x_{d_i} - x_{d_j}, \tag{26a}$$

$$\Delta Y_{ji} = \Delta y_{d_j d_i} + \rho_{d_j m}\sin(\lambda_{d_j m}), \qquad \Delta y_{d_j d_i} \triangleq y_{d_i} - y_{d_j}, \tag{26b}$$

and $x^R_{ji;k} = \begin{bmatrix} \Delta x_{d_j d_i} & \Delta y_{d_j d_i} \end{bmatrix}^T$. By combining the physical measurement model of Eq. (9), its health status defined in Eq. (10), and the indirect measurement model of Eq. (25), we can write the combined measurement model as

$$z_k \triangleq \begin{bmatrix} z_{d_1;k} \\ \vdots \\ z_{d_n;k} \end{bmatrix} = h(x_k, x^R_{dd;k}) + v_k = n_a^{-1}$$
$$\times \begin{bmatrix} h_{d_1}(x_k) & h^\dagger_{d_2}(x^R_{21;k}) & \dots & h^\dagger_{d_n}(x^R_{n1;k}) \\ h^\dagger_{d_1}(x^R_{12;k}) & h_{d_2}(x_k) & \dots & h^\dagger_{d_n}(x^R_{n2;k}) \\ \vdots & & \ddots & \vdots \\ h^\dagger_{d_1}(x^R_{1n;k}) & \dots & h^\dagger_{d_{n-1}}(x^R_{n-1n;k}) & h_{d_n}(x_k) \end{bmatrix} \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \\ \vdots \\ \varsigma_n \end{bmatrix} + v_k \tag{27}$$

where $x^R_{dd;k}$ is a vector containing the relative positions between all active defender pairs at time t_k, i.e., it contains $\Delta x_{d_j d_i}$ and $\Delta y_{d_j d_i}$, for all $i, j \in \{1, \dots, n\}$, $i \neq j$, $\varsigma_i \neq 0$, and $\varsigma_j \neq 0$. The functions h_{d_i} and $h^\dagger_{d_i}$ are defined in Eqs. (9) and (25), respectively, $z_{d_i;k}$ is the physical LOS angle measurement of the i-th defender, and $v_k \triangleq [v_{d_1;k} \quad \dots \quad v_{d_n;k}]^T$. Note, the argument x_k in $h^\dagger_{d_i}$ was omitted for notation simplicity. In the next subsection, the combined measurement model of Eq. (27) will be used to design the reduced-order estimator.

III.D. Extended Kalman Filter Design

As the estimation model of Eq. (18) is nonlinear, an extended Kalman filter (EKF) is used to estimate the state vector defined in Eq. (17). The state estimate of the filter at time t_k using measurements up to time t_{k-1}, $\hat{x}_{x|k-1}$, is propagated in time using Eq. (24) and the most up-to-dated $x^R_{td;k}$. The state transition matrix $\Phi_{k|k-1}$ associated with the system dynamics of Eq. (18) can be approximated by

$$\Phi_{k|k-1} = \exp(F_{k-1|k-1}T) \approx I + F_{k-1|k-1}T, \tag{28}$$

where $T = t_k - t_{k-1}$ is the sampling time used for time propagation, I is the identity matrix of appropriate dimension, and $F_{k-1|k-1}$ is the Jacobian matrix associated with the dynamics of Eq. (24), i.e.,

$$F_{k-1|k-1} = \left.\frac{\partial f_{k-1}(x, x^R_{td})}{\partial x}\right|_{x=\hat{x}_{k-1|k-1}}, \tag{29}$$

is assumed to be fixed during the time interval $(t_{k-1}, t_k]$. The prediction error covariance matrix is

$$P_{k|k-1} = \Phi_{k|k-1} P_{k-1|k-1} \Phi^T_{k|k-1} + Q_k, \tag{30}$$

where Q_k is a covariance matrix of the equivalent discrete process noise, i.e.,

$$Q_k = \int_0^T e^{F_{k-1|k-1}\eta} \Psi e^{F^T_{k-1|k-1}\eta} \mathrm{d}\eta, \tag{31}$$

where Ψ is a matrix whose only nonzero elements are $\Psi(i,i) > 0$, $i = 2n+1, \dots, 2n+n_m$. This matrix is used as a tuning matrix, see [37, 45] for more details.

The measurement update stage depends on whether the measurements and the relative geometries have been shared successfully or not. Therefore, we will first present the general equations when all the measurements are available, i.e., $n_a = n$, and then we will discuss the case when $n_a < n$.

The state estimate $\hat{x}_{k|k-1}$ is updated by

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left(z_k - h(\hat{x}_{k|k-1}, x^R_{dd;k})\right), \tag{32}$$

where $h(\cdot)$ is given in Eq. (27) and K_k is the Kalman gain computed as

$$K_k = P_{k|k-1} H_k^T \left(H_k P_{k|k-1} H_k^T + R\right)^{-1}, \tag{33}$$

where H_k is the measurement Jacobian matrix and R is the measurement noise covariance matrix

$$H_k = \left.\frac{\partial h(x, x^R_{dd;k})}{\partial x}\right|_{x=\hat{x}_{k|k-1}}, \quad R = \mathrm{diag}\left\{\sigma^2_{\lambda d_1 m} \quad \dots \quad \sigma^2_{\lambda d_n m}\right\}. \tag{34}$$

Finally, the covariance matrix is updated using

$$P_{k|k} = P_{k|k-1} - K_k H_k P_{k|k-1}. \tag{35}$$

and the measurement Jacobian matrix is given by

$$H_k = \left.\begin{bmatrix} \begin{matrix} H^{\rho}_{d_1 d_1} & \dots & H^{\rho}_{d_n d_1} & H^{\lambda}_{d_1 d_1} & \dots & H^{\lambda}_{d_n d_1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ H^{\rho}_{d_1 d_n} & \dots & H^{\rho}_{d_n d_n} & H^{\lambda}_{d_1 d_n} & \dots & H^{\lambda}_{d_n d_n} \end{matrix} & [0] \end{bmatrix}\right|_{x=\hat{x}_{k|k-1}}, \tag{36}$$

where $[0]$ is a matrix of zeros with dimension $n \times (3 + n_m)$, and

$$H^{\rho}_{d_j d_i} = \begin{cases} \dfrac{1}{n_a} \dfrac{\Delta x_{d_j d_i} \sin(\lambda_{d_j m}) - \Delta y_{d_j d_i} \cos(\lambda_{d_j m})}{\Lambda_{ji}} \varsigma_j, & i \neq j \\ 0, & i = j \end{cases}, \tag{37a}$$

$$H^{\lambda}_{d_j d_i} = \begin{cases} \dfrac{1}{n_a} \dfrac{\left(\Omega_{ji} + \rho_{d_j m}\right) \rho_{d_j m}}{\Lambda_{ji}} \varsigma_j, & i \neq j \\ 1, & i = j \end{cases}, \tag{37b}$$

with $\Omega_{ji} = \Delta x_{d_j d_i} \cos(\lambda_{d_j m}) + \Delta y_{d_j d_i} \sin(\lambda_{d_j m})$ and the common denominator Λ_{ji} is

$$\Lambda_{ji} = \Delta x^2_{d_j d_i} + \Delta y^2_{d_j d_i} + \rho^2_{d_j m} + 2\rho_{d_j m} \Omega_{ji}.$$

In Eq. (37), $\rho_{d_j m}$ and $\lambda_{d_j m}$ are replaced with the appropriate values from $\hat{x}_{k|k-1}$. The relative positions, $\Delta x_{d_j d_i}$ and $\Delta y_{d_j d_i}$, defined in Eq. (26), are considered from the k-th time frame.

If the physical measurement from the i-th defender, $z_{i;k}$, is not available (e.g., because the i-th defender ceased to exist, or due to sensor error, blind range of the sensor, etc.), then the i-th row from H_k, $h(\cdot)$, and z_k, respectively, and the i-th row and the i-th column from R_k are eliminated.

III.E. Comments on Implementation and Observability Issues

The reduced-order estimation scheme presented earlier allows to implement the algorithm on a single defender only (centralized approach). In such a case, at each computation cycle (time step k), the designated defender collects all the measurements from other missiles, acquires the relative states $x^R_{dd;k}$ and $x^R_{td;k}$, and subsequently computes the state estimate $\hat{x}_{k|k}$ and shares it with the other defenders.

The centralized approach allows to reduce the requirements on the on-board computers for the other $n-1$ defenders, because all the estimation-related computations are performed on a single on-board computer of the designated defender. Note, however, that in this formulation, the designated defender has to solve a higher dimension estimation problem, which might lead to higher computational burden than multiple lower-order estimation problems run by each defender separately (decentralized approach). Moreover, the centralized approach is more prone to failures as the defenders that do not run the estimator might lack the updated state estimates in case of communication problems.

If the reliability and robustness of the estimation algorithm is of the utmost importance, then each defender can compute its own joint state estimate $\hat{x}_{k|k}$. It can also share it with the rest of the team, and perform a cross-check (consistency check) with the other $n-1$ estimates. Such concept will lead to increased communication overhead and to higher computation effort of all n defenders.

From observability point of view, as only LOS angle measurements are available, the quality of the estimation depends upon the defenders' trajectories and hence on the implemented defenders' guidance law. Based on similar analysis as in [10], the i-th defender-missile range, $\rho_{d_i m}$, can be calculated using the noisy measurements of the i-th defender (z_{d_i}) and the j-th defender (z_{d_j}) as follows

$$\bar{\rho}_{d_i m} = \rho^d_{ji} \frac{\sin(\lambda^d_{ji} - z_{d_j})}{\sin(z_{d_i} - z_{d_j})}, \qquad i, j \in \{1, \dots, n\},\ i \neq j, \tag{38}$$

where

$$\rho_{ji}^d = \sqrt{\Delta x_{d_j d_i}^2 + \Delta y_{d_j d_i}^2}, \qquad \lambda_{ji}^d = \mathrm{atan2}\left(\Delta y_{d_j d_i}, \Delta x_{d_j d_i}\right), \tag{39}$$

and $\Delta x_{d_j d_i}$ and $\Delta y_{d_j d_i}$ are given in Eq. (26). The range $\bar{\rho}_{d_i m}$ can be viewed as a pseudomeasurement at time step k. It can be shown that $\bar{\rho}_{d_i m}$ has a non-stationary normal distribution, i.e.,

$$\bar{\rho}_{d_i m} \sim \mathcal{N}\left(\rho_{d_i m}, \sigma_{\bar{\rho} d_i m}^2\right), \tag{40}$$

where $\rho_{d_i m}$ is the true range and $\sigma_{\bar{\rho} d_i m}$ is the standard deviation defined as

$$\begin{aligned}\sigma_{\bar{\rho} d_i m} &= \rho_{ji}^d \\ &\times \frac{\sqrt{\sin^2(\lambda_{ji}^d - \lambda_{d_i m})\sigma_{\lambda d_j m}^2 + \sin^2(\lambda_{ji}^d - \lambda_{d_j m})\cos^2(\Delta\lambda_{ij})\sigma_{\lambda d_i m}^2}}{\sin^2(\Delta\lambda_{ij})},\end{aligned} \tag{41}$$

with $\Delta\lambda_{ij} \triangleq \lambda_{d_i m} - \lambda_{d_j m}$. From Eq. (41), it can be concluded that if the difference between the LOS angle of the i-th defender and the j-th defender, i.e., $|\Delta\lambda_{ij}|$, becomes small (close to zero), the variance of $\bar{\rho}_{d_i m}$ increases, which in turn may cause significant deterioration in the estimation accuracy, especially in the range, missile's speed, and time-to-go estimates. Consequently, poor estimation performance might lead to poor guidance performance.

When using multi LOS angle measurements only, the above analysis suggests that in order to achieve good overall state estimation performance, $|\Delta\lambda_{ij}|$ needs to be kept far from zero throughout the engagement for all $i \neq j$. In other words, one must ensure trajectory separation between the defenders. In the next section, we present a guidance strategy which can enforce a specified relative intercept angle between two successive defenders. As will be shown in Sec. V, carefully selected nonzero angle constraints for the consecutive pairs of defenders, can naturally lead to trajectory separation between them.

Remark 4. As for any information sharing estimation concept, a possible implementation challenge might be the delayed arrival of data (measurements and/or relative positions). Yet, as long as the internal clocks of the defenders are accurately synchronized and the data are sent with an accurate time stamp, then the delayed information can be easily incorporated into the estimator using estimation techniques developed for state estimation with delayed measurements [46, 47]. However, this issue has typically a smaller effect on the performance than communication problems or jamming [34].

IV. Cooperative Guidance Law for the Defenders

In this section, we will discuss a cooperative guidance law for the team of defenders, which can ensure trajectory separation between the defenders. This guidance law exploits an explicit cooperation of the defenders to impose an angular geometry at the point of intercept and an implicit cooperation between the target and the defenders. The implicit cooperation of the target stems from the fact that the defenders are aware of the future maneuver of the target and thus can anticipate the maneuvers it will induce on the incoming homing missile.

IV.A. Relative Intercept Angle Guidance

Here, following the exposition in [34], we briefly recall the recently developed cooperative optimal guidance law for imposing a relative geometry in between a group of missiles and a single moving target at intercept, while minimizing the expected miss distance and the control effort of the missiles. The problem was posed in the LQ optimal control framework, and solutions were obtained for any team size with any linear missile dynamics. The guidance law was derived under the assumption of linear kinematics, perfect information, and unbounded controls. For our target-defenders-missile scenario, we will use a slightly modified version of this guidance law to enhance estimation by imposing nonzero relative intercept angles between consecutive defenders. Some assumptions made in [34] are also relaxed.

Let us denote the angle between the i-th defender and the missile as $\gamma_{d_i m} = \gamma_{d_i} + \gamma_m$. The difference between the intercept angles $\gamma_{d_i m}$ and $\gamma_{d_{i+1} m}$ is the relative intercept angle from the missile's perspective. This is the angle that will be enforced by the presented guidance law, see Fig. 2 which depicts the relevant

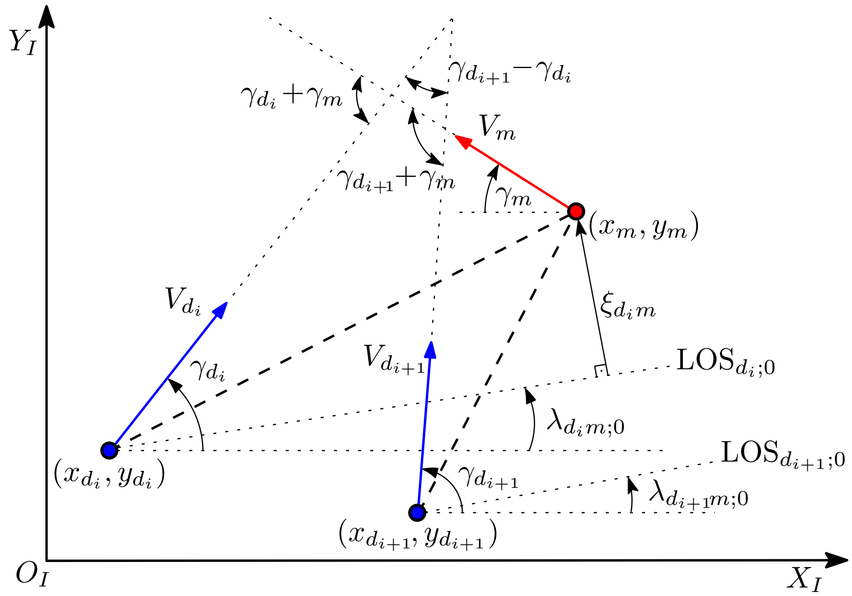


Figure 2: Relative angular geometry and linearization parameters.

relative angles and linearization parameters used within the guidance law. The optimal closed-loop guidance law of the i-th defender was found to have the following form

$$
\begin{aligned}
u_{d_i}(t) &= \sum_{j=1}^{n} \frac{N_{Z_j}^{u_i}}{(t_{d_i m}^{go})^2} Z_j(t) \\
&+ \sum_{j=1}^{n-1} N_{\Delta Z_{n+j}}^{u_i} \frac{V_{d_i}}{t_{d_i m}^{go}} \left(Z_{n+j}(t) - Z_{n+j+1}(t) - \Delta_{c_j} \right), \quad t \in [0, t_{d_i m}^{f}],
\end{aligned} \tag{42}
$$

where the navigation gains $N_{Z_j}^{u_i}$ and $N_{\Delta Z_{n+j}}^{u_i}$ are functions of $t_{d_l m}^{go}$, $l \in \{1, \ldots, n\}$ and are given in [34] for the general n missile case. For ideal defenders' dynamics (i.e., zero lag), the ZEM distances, Z_i, $i = 1, \ldots, n$, and the zero-effort flight-path angles (ZEA-s) of the missile plus that of the particular defender, Z_{n+i}, $i = 1, \ldots, n$, are give by [34]

$$Z_i(t) = \xi_{d_i m} + \dot{\xi}_{d_i m} t_{d_i m}^{go} + k_{d_i m} a_m (t_{d_i m}^{go})^2 / 2, \tag{43a}$$

$$Z_{n+i}(t) = \gamma_{d_i} + \gamma_m + t_{d_i m}^{go} a_m / V_m, \tag{43b}$$

where $\xi_{d_i m}$ is the relative displacement between the missile and the i-th defender normal to the LOS used for linearization, denoted as $\mathrm{LOS}_{i;0}$. In Eq. (42), Δ_{c_i} represents the required relative intercept angle between the i-th and the $(i+1)$-th defender. The linearization parameter $k_{d_i m}$ satisfies

$$k_{d_i m} = \cos(\gamma_{m;0} + \lambda_{d_i m;0}), \qquad i \in \{1, \ldots, n\}, \tag{44}$$

and relates the missile acceleration a_m to the direction normal to $\mathrm{LOS}_{i;0}$. In Eq. (44), $\gamma_{m;0}$ is the initial flight path angle of the missile and $\lambda_{d_i m;0}$ is the i-th defender LOS angle used for linearization.

The cooperative guidance law of Eq. (42) minimizes the following cost function

$$
\begin{aligned}
J = &\frac{\alpha_1}{2} \xi_{d_1 m}^2 (t_{d_1 m}^{f}) + \ldots + \frac{\alpha_n}{2} \xi_{d_n m}^2 (t_{d_n m}^{f}) \\
&+ \frac{\beta_1}{2} \left(\gamma_{d_1 m}(t_{d_1 m}^{f}) - \gamma_{d_2 m}(t_{d_2 m}^{f}) - \Delta_{c_1} \right)^2 + \ldots \\
&+ \frac{\beta_{n-1}}{2} \left(\gamma_{d_{n-1} m}(t_{d_{n-1} m}^{f}) - \gamma_{d_n m}(t_{d_n m}^{f}) - \Delta_{c_{n-1}} \right)^2 \\
&+ \frac{1}{2} \int_0^{t_{d_1 m}^{f}} \eta_1^2 u_{d_1}^2 \mathrm{d}t + \ldots + \frac{1}{2} \int_0^{t_{d_n m}^{f}} \eta_n^2 u_{d_n}^2 \mathrm{d}t,
\end{aligned} \tag{45}
$$

where α_i, η_i, and β_i are nonnegative weights. This cost function enforces an explicit cooperation between the defenders, as their trajectories are mutually dependent on each other. Letting $\alpha_i \to \infty$ yields a perfect intercept between the i-th defender and the missile. Similarly, letting $\beta_i \to \infty$ enforces perfect intercept angle Δ_{c_i} between successive defenders. The parameter η_i weights the i-th defender's control effort.

The guidance law of Eq. (42) was derived under the assumption that the future missile maneuver as well as the relative states are known or are accurately measured. This assumption can be directly relaxed by making use of the proposed estimation concept presented in Sec. III. However, the ZEM distances and the ZEA-s of Eq. (43) are valid only under the assumption that the missile maintains a known constant maneuver throughout the engagement, i.e., $u_m(t) = const., \forall t \geq 0$. It is apparent that for our case this assumption is likely to be violated as the missile is homing onto the target using one of the guidance laws presented in Sec. III.A and the target is performing some sort of (evasive) maneuvers. Next, we will discuss the possible guidance strategies of the target and we will present the necessary modifications of the above guidance law to cope with the maneuvering missile problem.

IV.B. Target's Guidance and its Implicit Cooperation

For sake of generality, in this paper, we assume that the guidance strategy of the target is arbitrary. The only assumption that we impose is that the defenders are fully aware of the future maneuvers of the target. Examples of possible design formulations of the target's guidance strategy are:

1. The target may perform a constant maximum acceleration maneuver to one side or an optimally adjusted evasion maneuver from the homing missile. For the missile guidance laws presented in Sec. III.A and for bounded target acceleration, the resulting optimal maneuvers have a bang-bang structure [16].

2. Obviously, the optimal evasion strategy suggested in the previous point is not necessarily the "best" from the defenders' perspective. Exploiting the fact that the missile is homing onto the target and that the target's guidance strategy directly shapes the missile's trajectory, which in turn indirectly influences the trajectories of the defenders, the target's guidance law can be design such that the defenders' control effort is minimized. By doing so, the target can lure in the attacking missile in regions where significantly less maneuverability is required from the defenders to hit the missile under a predefined relative intercept angle. This problem was addressed in [30] for a single defender scenario.

3. Last but not least, the target's guidance law can be design by combining the previous two approaches and/or by shaping the missile's and defenders' trajectories into regions with added information content, see for instance [5, 48, 49].

Remark 5. The assumption on the known target's future maneuvers implies that the target is expected to pursue the evasive strategy that was communicated to the defenders. In case of a manned aircraft this means that the pilot will generate evasion flight trajectories corresponding to the communicated evasion strategy. For a UAV, full autonomy is often required, thus an autoevasive autopilot is a natural choice.

For a given missile guidance strategy, the information about the target's future maneuvers can be very helpful to obtain the missile acceleration profile as a function of time, i.e., $a_m(t)$. This can be achieved via numerical integration of the appropriate engagement equations. By doing so, we can relax the constant missile acceleration assumption in Eq. (43). Additionally, the potential intercept points of the defenders can be predicted, which in turn can help to reduce the defenders' acceleration demand, hence reduce the likelihood of control saturation.

Let us assume for a while that the future missile maneuver is known and it is not constant, then based on the terminal projection transformation, the zero-effort variables of Eq. (43) are generally given by [33, 50]

$$Z(t) = D\Phi^{(d_i)}(t^{go}_{d_i m})x^{(d_i)}(t) + D \int_t^{t^f_{d_i m}} \Phi^{(d_i)}(t^f_{d_i m}, \tau)C^{(d_i)}u_m \mathrm{d}\tau, \tag{46}$$

where $i \in \{1, \dots, n\}$, D is a constant row vector that pulls out the appropriate element of the zero-effort variable, $\Phi^{(d_i)}$ is the transition matrix associated with the homogenous solution of the linearized i-th defender-missile engagement, and $C^{(d_i)}$ is a vector associated with the linearized one-sided problem, see [34] for more details. Using Eq. (46) and replacing the LOS used for linearization in Eq. (44) with the instantaneous LOS, i.e., $k_{d_i} = \cos(\gamma_{d_i} - \lambda_{d_im})$, the computation of the ZEM distances and ZEA-s of Eq. (43) can be replaced by the following equations (assuming constant missile speed V_m)

$$Z_i(t) = \xi_{d_im} + \dot{\xi}_{d_im} t^{go}_{d_im} + \int_t^{t^f_{d_im}} (t^f_{d_im} - \tau) a_m(\tau) \cos\big(\gamma_m(\tau) + \lambda_{d_im}(\tau)\big) \mathrm{d}\tau, \tag{47a}$$

$$Z_{n+i}(t) = \gamma_{d_i} + \gamma_m + \frac{1}{V_m} \int_t^{t^f_{d_im}} a_m(\tau) \mathrm{d}\tau, \tag{47b}$$

where the integral components in Eq. (47) are computed by numerical integration and time propagation of the relevant parts of Eqs. (2) and (4), assuming that no further acceleration commands are issued by the defenders and that the missile and the target follow the presumed maneuvering model. The time propagation can be performed e.g., by using a fourth-order Runge-Kutta (4RK) algorithm and the numerical integration can be computed by any suitable numerical integration technique, e.g., trapezoidal rule. The integration is performed using a constant number of integration steps from t to $t^f_{d_im}$. Consequently, the computation complexity at each time step is equal and the integration resolution improves as the defenders and the missile approach each other [30]. Note, if a_m is constant throughout the engagement, then Eq. (47) degenerates to Eq. (43).

Remark 6. To avoid the constant acceleration assumption and to improve accuracy, an alternative approach known as "predictive guidance" can be considered [51]. The idea is to compute Z_i, $i = 1, \ldots, 2n$ by integrating the appropriate engagement's equations from t to $t^f_{d_im}$ at each time step with nulled defenders' controls and presumed target's and missile's maneuvers. Then, by definition, Z_i, $i = 1, \ldots, n$ are the miss distances obtained by this simulation. Similarly, Z_{n+i}, $i = 1, \ldots, n$ are the intercept angles between the particular defender and the missile obtained from the simulation. This approach tends to yield a more accurate evaluation of the ZEM and the ZEA, which can be directly used in the defenders' guidance law implementation. Nevertheless, this approach requires considerably more computational effort as computation of Eq. (47).

IV.C. Implementation Issues

The variables ξ_{d_im} and $\dot{\xi}_{d_im}$, which appear in Z_i, $i = 1, \ldots, n$ of Eq. (47a), relate to the linearized model. To implement the defenders' guidance law in a nonlinear setting, we need to replace these variables by more meaningful kinematic variables. Assuming small deviations from collision triangle, the displacement ξ_{d_im} can be reasonably well approximated by

$$\xi_{d_im} \approx \rho_{d_im}(\lambda_{d_im} - \lambda_{d_im;0}). \tag{48}$$

Differentiating Eq. (48) with respect to time yields

$$\xi_{d_im} + \dot{\xi}_{d_im} t^{go}_{d_im} = -V_{\rho d_im} \dot{\lambda}_{d_im} (t^{go}_{d_im})^2. \tag{49}$$

The left hand side of Eq. (49) is identical to the first two terms of Eq. (47a), respectively. Therefore, using Eq. (49), ξ_{d_im} and $\dot{\xi}_{d_im}$ can be replaced by $\dot{\lambda}_{d_im}$ and $V_{\rho d_im}$, defined in Eqs. (2) and (3a), respectively.

Due to the same assumption, the speed $V_{\rho d_im}$ can be assumed constant, and the $t^{go}_{d_im}$, defined in (7), can be approximated by

$$\bar{t}^{go}_{d_im} \approx \begin{cases} -\rho_{d_im}/V_{\rho d_im}, & V_{\rho d_im} < 0 \\ 0, & V_{\rho d_im} \geq 0 \end{cases}, \quad i \in \{1, \ldots, n\}. \tag{50}$$

Note that in some cases, a more accurate $t^{go}_{d_im}$ estimate might be needed as the one given in Eq. (50), see for instance the one reported in [32, 52]. Now, all variables needed for the proper implementation of the defenders' guidance law given by Eq. (42) are either part of the joint defenders' state estimate $\hat{x}_k$ (see the definition in Eq. (17)) or they are assumed to be known to high accuracy (V_i and γ_i for $i \in \{t, d_1, \ldots, d_n\}$).

As outlined in Section III.C, the defenders' measurements $z_{d_i;k}$ are acquired and shared (without any delay) at discrete time instances $t_k = k \cdot T$. Similarly, execution of any computer-based algorithm is performed in discrete time intervals. Therefore, the defenders' control commands $u_{d_i}(t)$, $i = 1, \ldots, n$ are assumed to be computed and executed at discrete time $t = t_k \triangleq k \cdot T_c$, where the flight computer's computational cycle $T_c > 0$ is assumed to be constant and equal to the sampling rate of the measurements, i.e., $T_c = T$.

V. Simulation Study

In this section, we analyze the proposed cooperative estimation/guidance strategy using numerical simulations. Our paramount interest is to study the effect of different values of the commanded relative intercept angle: a) on the pure estimation performance, and b) on the intertwined guidance-estimation performance while also considering different maneuverability requirements for the defenders.

First, we present the considered engagement scenario and simulation parameters, followed by a sample run demonstration. Finally, we present two Monte Carlo (MC) studies, one evaluating the pure estimation performance in open loop, and the second evaluating in closed loop the intertwined guidance-estimation performance in terms of the achieved miss distance, acceleration requirement, and intercept angle precision.

V.A. Considered Engagement and Simulation Parameters

For simulation purposes, we consider two defending missiles ($n = 2$). Both defenders are launched from the targeted aircraft at the beginning of the engagement. The horizontal separation between the target and the missile is 5 [km]. The defenders are initiated at a vertical separation of $\Delta y_{td_1} = \Delta y_{td_2} = -1$ [m] below the target. The target's speed is $V_t = 300$ [m/s] and the speed of the two defenders and the missile is equal, i.e., $V_{d_1} = V_{d_2} = V_m = 500$ [m/s]. We consider the missile and the target having first-order strictly proper dynamics with time constants $\tau_m = 0.2$ [s] and $\tau_t = 0.5$ [s], respectively. Thus, matrices in Eq. (4) degenerate to $A_i = -1/\tau_i$, $B_i = 1/\tau_i$, $C_i = 1$, and $D_i = 0$, $i \in \{m, t\}$. We consider ideal dynamics for the defenders. The missile's initial flight path angle is chosen such that the missile's velocity vector points towards the initial target location, i.e., $\gamma_{m;0} = 0$ [deg]. As the defenders are launched from the aircraft's platform, therefore the initial flight path angles of the defenders are considered to be identical to the initial flight path angle of the target, i.e., $\gamma_{d;0} = \gamma_{t;0}$. For the closed loop MC analysis, these angles are drawn from a uniform distribution on the interval $[-30, 30]$ [deg]. The target's maneuverability is limited to $u_t^{max} = 5$ [g], where g = 9.80665 [m/s^2] is the standard acceleration due to the gravity. The commanded relative intercept angle $\Delta_c \triangleq \Delta_{c_1}$ and the defenders' maneuverability limits u_d^{max} belong to closed sets $\mathcal{D}_c$ and $\mathcal{U}_d$, defined as

$$\mathcal{D}_c \triangleq \{5, 10, \ldots, 150\} \text{ [deg]}, \qquad \mathcal{U}_d \triangleq \{10, 20, 30, 40, \infty, \infty^\star\} \text{ [g]},$$

where $\infty^\star$ represents a special case when, in addition to the lacking maneuverability limitation, the defenders are also assumed to have perfect information about the missile and engagement parameters, i.e., noise-free and without an estimator in the guidance loop. In the other cases (i.e., cases without the star), the defenders are guided using estimated states. Note that the omitted subscript "i" for u_d^{max} indicates that both defenders are equally concerned, i.e., $u_d^{max} = u_{d_1}^{max} = u_{d_2}^{max}$. In the rest of the paper, we will use this type of notation for other variables too.

In all considered simulations, the missile employs PN guidance law with $N' = 4$. This guidance law is implemented without an estimator in the loop (perfect information) and without any maneuverability limits, i.e., $u_m^{max} = \infty^\star$ [g]. Similarly, in all simulations, the target applies a constant maximum acceleration maneuver to one side. The maneuver direction is chosen based on the engagement's initial geometry as

$$u_t = \begin{cases} +u_t^{max} & \text{if} \quad \gamma_{t;0} \geq 0, \\ -u_t^{max} & \text{if} \quad \gamma_{t;0} < 0, \end{cases}$$

where $\gamma_{t;0}$ is the initial flight path angle of the target. The defenders employ the cooperative guidance law of Eq. (42) with implicit target cooperation. Based on the discussions presented in Sec. IV.B, the guidance law of Eq. (42) for $n = 2$ degenerates to

$$u_{d_1} = \frac{N_{Z_1}^{u_1}}{(\bar{t}_{d_1m}^{go})^2} Z_1 + \frac{N_{Z_2}^{u_1}}{(\bar{t}_{d_1m}^{go})^2} Z_2 + N_{\Delta Z_3}^{u1} \frac{V_{d_1}}{\bar{t}_{d_1m}^{go}} \big(Z_3 - Z_4 - \Delta_c\big),$$

$$u_{d_2} = \frac{N_{Z_1}^{u_2}}{(\bar{t}_{d_2m}^{go})^2} Z_1 + \frac{N_{Z_2}^{u_2}}{(\bar{t}_{d_2m}^{go})^2} Z_2 + N_{\Delta Z_3}^{u2} \frac{V_{d_2}}{\bar{t}_{d_2m}^{go}} \big(Z_3 - Z_4 - \Delta_c\big),$$

where the navigation gains and the zero-effort variables are given in the Appendix. The numerical values of the defenders' guidance parameters are chosen as: $\alpha_1 = \alpha_2 = 10^5$, $\beta_1 = 10^8$, and $\eta_1 = \eta_2 = 1$.

The estimator developed in Sec. III is implemented at a sampling rate of 50 [Hz] ($T = 1/50$ [s]). The simulated measurement noises are with $\sigma_{\lambda dm} = 1$ [mrad]. The filter's tuning parameter Ψ has been chosen by numerical simulations. The initial state of the filter is sampled from a Gaussian distribution, i.e.,

$$\hat{x}_{0|0} \sim \mathcal{N}(x_0, P_{0|0}),$$

where x_0 is the true state vector and $P_{0|0}$ is the initial covariance matrix of the error given by

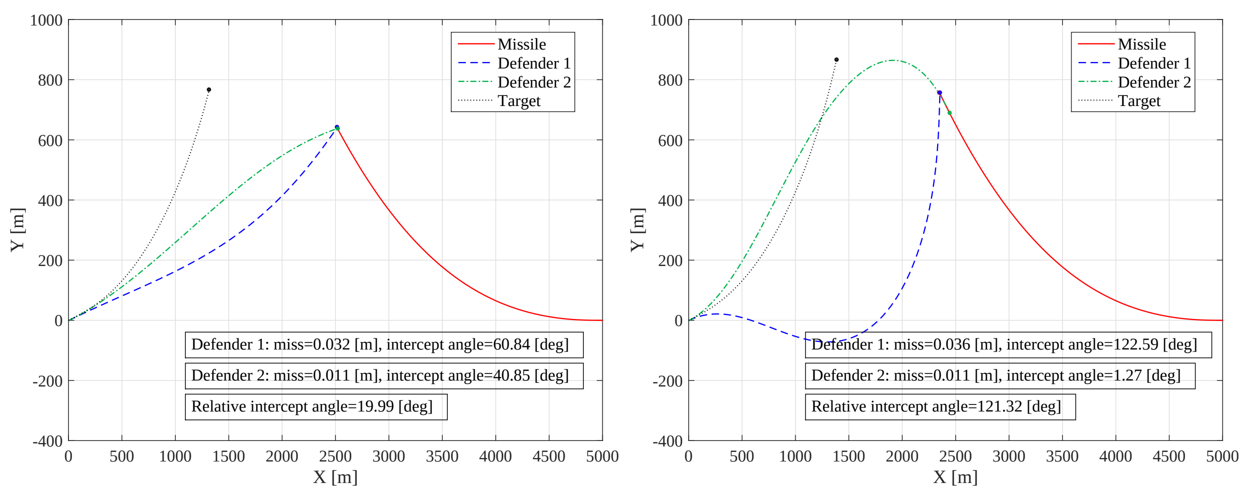
$$\sqrt{P_{0|0}} = \text{diag}\left\{100 \quad 100 \quad 5\pi/180 \quad 5\pi/180 \quad 2.5\text{g} \quad 5\pi/180 \quad 50 \quad 2\right\}.$$

The nonlinear equations of motion of the target-defenders-missile engagement are solved using the 4RK algorithm. To ensure precise evaluation of the terminal guidance performance, high resolution integration is performed when the defenders are close to the missile. After the leading defender has passed the missile, the simulation continues to run in order to evaluate the performance of the second defender.

V.B. Sample Run Example

Before turning to a statistical MC evaluation, first we demonstrate two sample runs for two different commanded relative intercept angles, namely for $\Delta_c = 20$ [deg] and for $\Delta_c = 120$ [deg]. The initial flight path angle of the target-defender team is $\gamma_{t;0} = \gamma_{d;0} = 10$ [deg] in both examples. The defenders are guided towards the missile using perfect information and with no maneuverability limitations, i.e., $u_d^{max} = \infty^\star$.

Figure 3 and 4 present the planar trajectories and the acceleration profiles of the target, missile, and the two defenders in the simulated sample runs, respectively. Figure 3 also contains the achieved miss distances and relative intercept angles for the considered runs. It can be seen from Fig. 4a that, although there is a requirement on a specific intercept angle of $\Delta_c = 20$ [deg] (achieved with 0.01 [deg] error), the maximal acceleration requirement from the defenders is quite small, approx. 6 [g], compared to the missile's maximal acceleration being above 7 [g]. On the other hand, as seen in Fig. 4b, significantly larger relative intercept angle requirement naturally leads to much higher maneuverability requirements from the defenders.



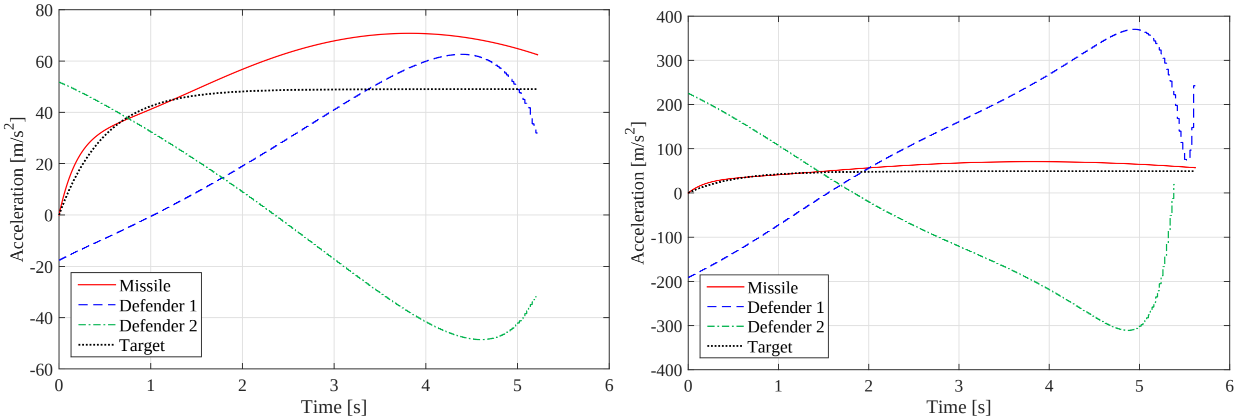
(a) $\Delta_c = 20$ [deg]. (b) $\Delta_c = 120$ [deg].

Figure 3: Sample trajectories for two different relative intercept angles.

V.C. Pure Estimation Performance Evaluation

Here, we evaluate the estimation performance of the proposed reduced-order estimation scheme for different relative intercept angles $\Delta_c \in \mathcal{D}_c$. To evaluate the estimation performance in open loop, we consider again perfect states and no saturation for the defenders, i.e., $u_d^{max} = \infty^\star$. For each value of Δ_c, a set of 500 MC runs was performed.

Note that if $\Delta_c = 0$ [deg], the resulting trajectories of the two defenders are similar. In such a case both defenders measure the same quantity (same look angle) and, as discussed in Sec. III.E, the triangulation



(a) $\Delta_c = 20$ [deg]. (b) $\Delta_c = 120$ [deg].

Figure 4: Sample acceleration profiles for two different relative intercept angles.

technique fails, the range, missile's speed, and time-to-go become weekly observable, and the filter diverges. For clarity of presentation, we have omitted the case of $\Delta_c = 0$ [deg] from our analysis.

Figure 5 shows the estimation performance for a particular intercept angle of $\Delta_c = 20$ [deg]. This angle corresponds to the one selected in the sample run analysis in the previous subsection. In addition to the estimated states, we also depict the error between $\bar{t}^{go}_{d_im}$ computed using true states and $\bar{t}^{go}_{d_im}$ computed estimated states. We denote this error as $\tilde{t}^{go}_{d_im}$. It can be observed Fig. 5, that despite the relatively small separation between the two defenders (see Fig. 3a), the estimator performs reasonably well and the standard deviations of the errors (actual σ) are rather consistent with those predicted by the filter.

Figure 6 shows the estimation performance for various values of Δ_c. The scalar measure used to compare the estimation performances is the actual standard deviation of the estimation errors (in Fig. 5 the line denoted as "actual σ"), evaluated at two different time instances. One being two seconds and the other being one second prior to the termination of the leading defender. It can be seen from Fig. 6 that for small Δ_c the estimation performance is very poor. Especially notice the case of $\Delta_c = \{5, 10\}$ [deg] for γ_m^{err} when the actual σ of the error is higher at $t^{go}_{dm} = 1$ [s] than at $t^{go}_{dm} = 2$ [s]. This suggest that the estimate of γ_m diverges for $\Delta_c = \{5, 10\}$ [deg]. In general, as the value of Δ_c increases, the estimation performance improves. Note the fluctuation in $\sigma(\lambda^{err}_{d_im})$. This phenomenon can be explained by the fact that the variables λ_{d_1m} and λ_{d_2m} are directly measured and the dynamics of the estimator does not have a significant effect on them. On the other hand, the magnitude of $\sigma(\lambda^{err}_{d_im})$ is actually smaller than the standard deviation of the measurement noise.

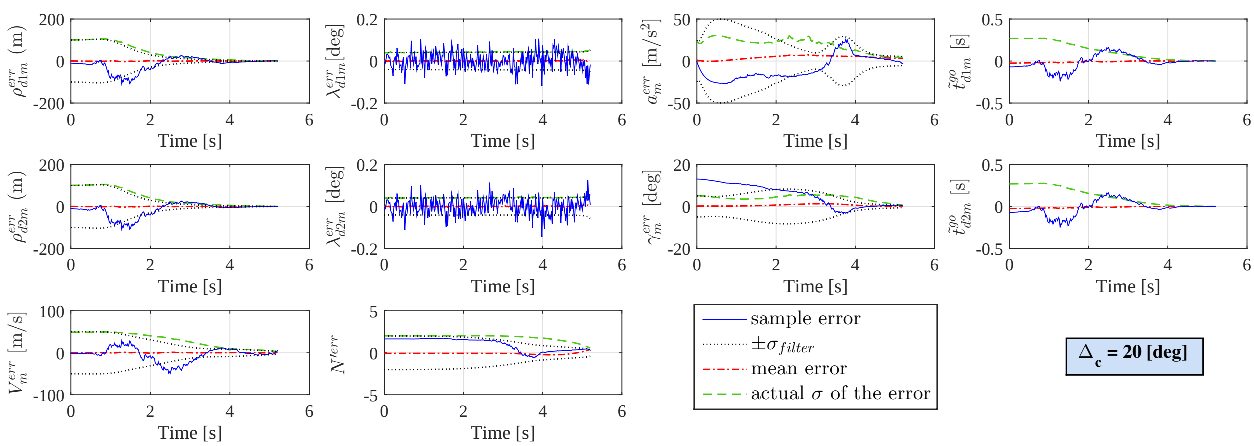


Figure 5: Estimation performance for a particular $\Delta_c = 20$ [deg].

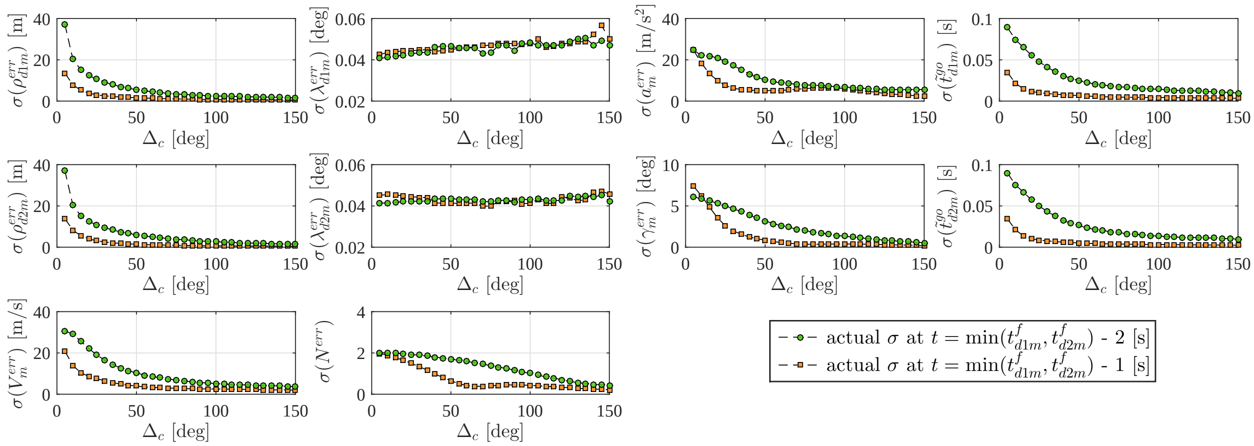


Figure 6: Estimation performance as a function of $\Delta_c \in \mathcal{D}_c$.

V.D. Intertwined Guidance-Estimation Performance Evaluation

The effect of different values of $\Delta_c \in \mathcal{D}_c$ on the intertwined guidance-estimation problem is analyzed here in closed loop. The analysis is done for various considerations of the defenders' maneuverability limit, i.e., $u_d^{max} \in \mathcal{U}_d$. For each value of Δ_c and u_d^{max}, a set of $n_{mc} = 500$ MC simulations was run, i.e., in total $\dim(\mathcal{D}_c) \times \dim(\mathcal{U}_d) \times n_{mc}$ runs. The guidance performance for each MC campaign is evaluated in terms of the achieved miss distances, defenders' acceleration requirements, and relative intercept angle errors.

For the miss distance evaluation, we first compute the "two defender" cumulative distribution function (CDF) which is defined on the minimum miss of both defenders. Then, using the obtained CDF, we compute the value of the miss which corresponds to the 95% of cases. This value is denoted as $\text{miss}_{95\%}$ and is mathematically given by

$$\text{Prob}\Big(\min_{i\in\{1,\dots,n_{mc}\}}\Big\{\rho^{(i)}_{d_1m}(t^f_{d_1m}), \rho^{(i)}_{d_2m}(t^f_{d_2m})\Big\} \leq \text{miss}_{95\%}\Big) = 0.95.$$

where the superscript (i) denotes the i-th MC realization. The quantity $\text{miss}_{95\%}$ is also known as "warhead lethality range" ensuring a 95% kill probability for the defenders team. To evaluate the maneuverability requirements, we consider the value of the two defender maximal acceleration in 95% of the simulation campaign cases. We denote this value as $a_d^{max}(95\%)$. This value is computed analogously as $\text{miss}_{95\%}$ is computed. Additionally to $a_d^{max}(95\%)$, we also consider a running cost J_{acc} on the acceleration profiles defined as

$$J_{acc} = \int_0^{t^f_{d_1m}} |a_{d_1}(\tau)|\mathrm{d}\tau + \int_0^{t^f_{d_2m}} |a_{d_2}(\tau)|\mathrm{d}\tau.$$

Figure 7 presents the obtained CDFs of the miss for $u_d^{max} \in \mathcal{U}_d$ and a particular relative intercept angle of $\Delta_c = 20$ [deg]. Note that the x-axis in Fig. 7 uses a logarithmic scale. The results show that with decreasing maneuverability the guidance performance deteriorates. As expected, the performance of any perfect information guidance law is better than the performance of the same guidance law using estimated states, see the results for $u_d^{max} = \{\infty, \infty^\star\}$ [g].

In Fig. 7, using different markers, we also depicted the values of $\text{miss}_{95\%}$ on the respective CDFs. These markers serve as building blocks in Fig. 8 which depicts the obtained results for all the considered intercept angles $\Delta_c \in \mathcal{D}_c$ and acceleration limits $u_d^{max} \in \mathcal{U}_d$. Note that the y-axis in Fig. 8 uses a logarithmic scale. Before commenting on Fig. 8, the results of Fig. 9 need to be introduced. Fig. 9a shows the control effort of the defenders in terms of the $a_d^{max}(95\%)$ measure while Fig. 9b in terms of the running cost J_{acc} measure.

Results presented in Figs. 6, 8, and 9 suggest, except for the case of perfect information and unbounded control, that smaller values of Δ_c yield to large miss distances due to the defenders' control saturation and poor estimation performance. From Fig. 10 it can be observed that the "overall" maneuverability (represented by J_{acc}) increases linearly with Δ_c while the "momentary" maneuverability (represented by a_d^{max}) reassembles a convex function for $u_d^{max} \neq \infty^\star$. The later can be explained by bad estimation accuracy for small values of Δ_c (see Fig. 6) and by the fact that large values of Δ_c require substantially more agility

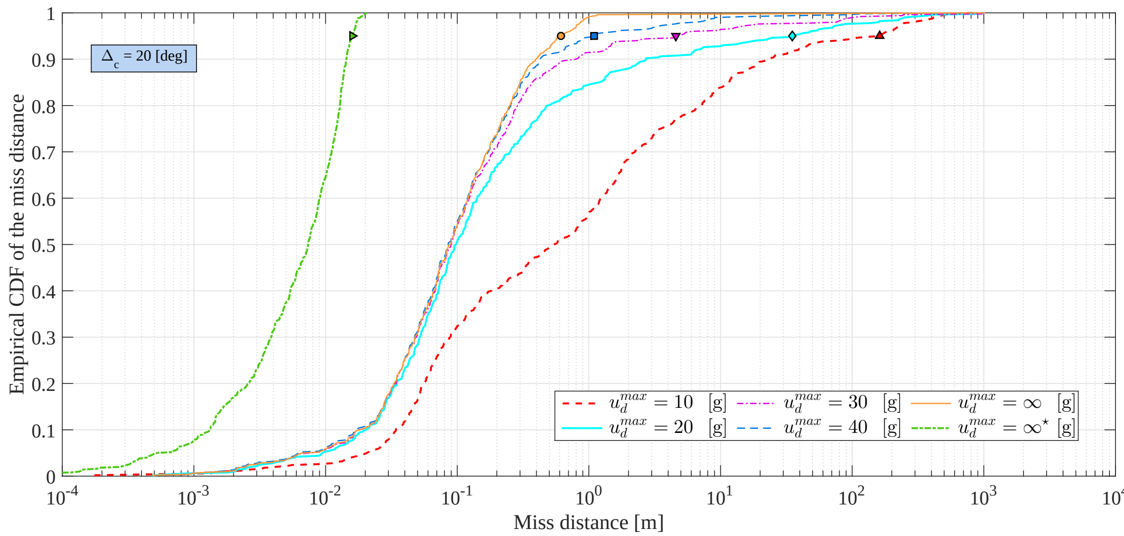


Figure 7: Empirical CDFs of the miss for $u_d^{max} \in \mathcal{U}_d$ and a particular $\Delta_c = 20$ [deg].

from the defenders. Consequently, bad estimation accuracy (resulting from small values of Δ_c) may cause control saturation, which in turn leads to larger misses. Note, however, that the control saturation is not the only reason for large misses for small Δ_c. The estimation accuracy also plays an important role, see the behaviour of $u_d^{max} = \infty$ [g] for $\Delta_c \in \{5, \dots, 40\}$ [deg] in Fig. 8. On the other hand, despite good estimation accuracy, limiting the defenders' maneuverability (u_d^{max}) worsens the target's protection capabilities if large intercept angles Δ_c are prescribed, see Fig. 8 for $\Delta_c \in \{70, \dots, 150\}$ and $u_d^{max} \neq \infty^\star$. Notice that for finite maneuverability limits ($u_d^{max} < \infty$), there exist a plateau effect, i.e., a region of intercept angles Δ_c, where the obtained miss is minimal. It is important to note that the defenders' control effort could be further reduced by an appropriate design of the target's guidance, see the discussion in Sec. IV.B.

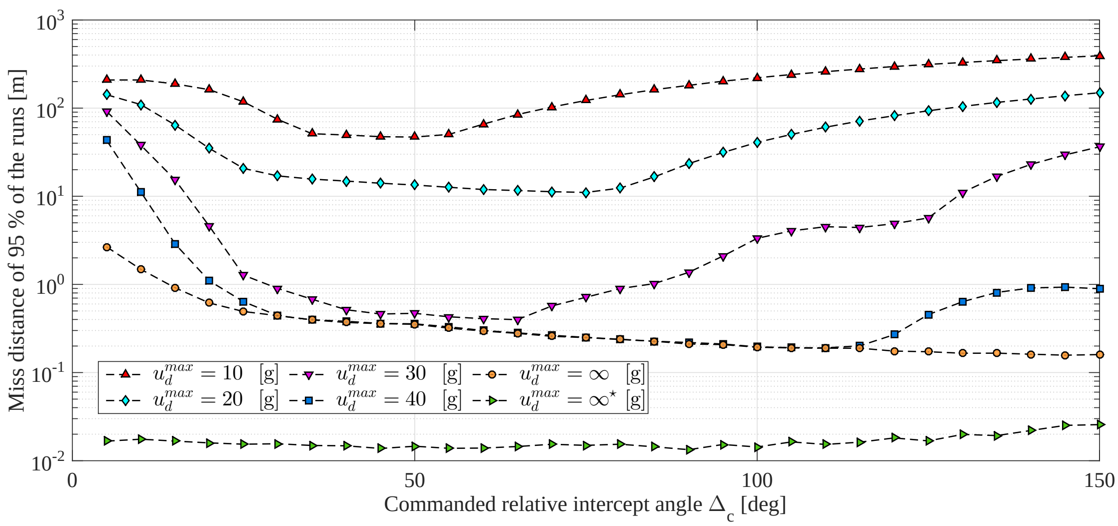
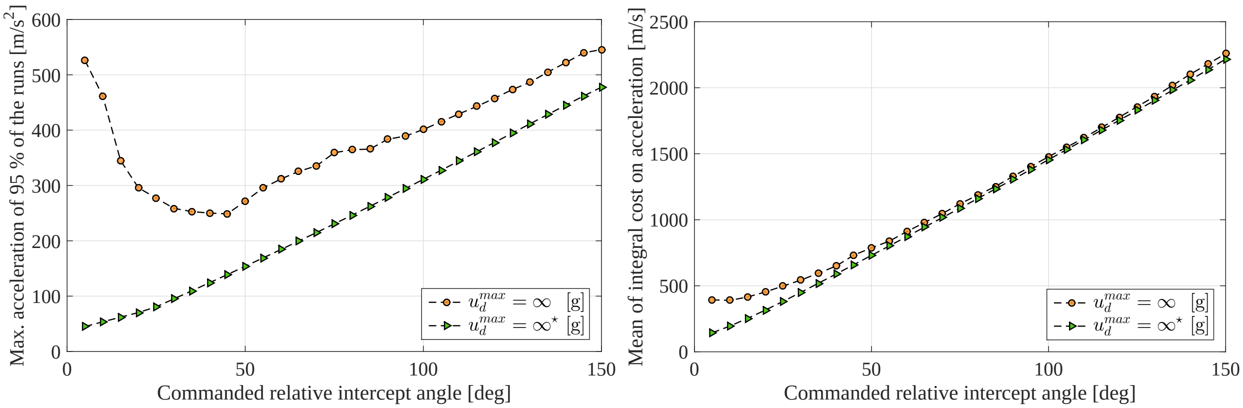


Figure 8: Values of $\text{miss}_{95\%}$ for $u_d^{max} \in \mathcal{U}_d$ as a function of $\Delta_c \in \mathcal{D}_c$.

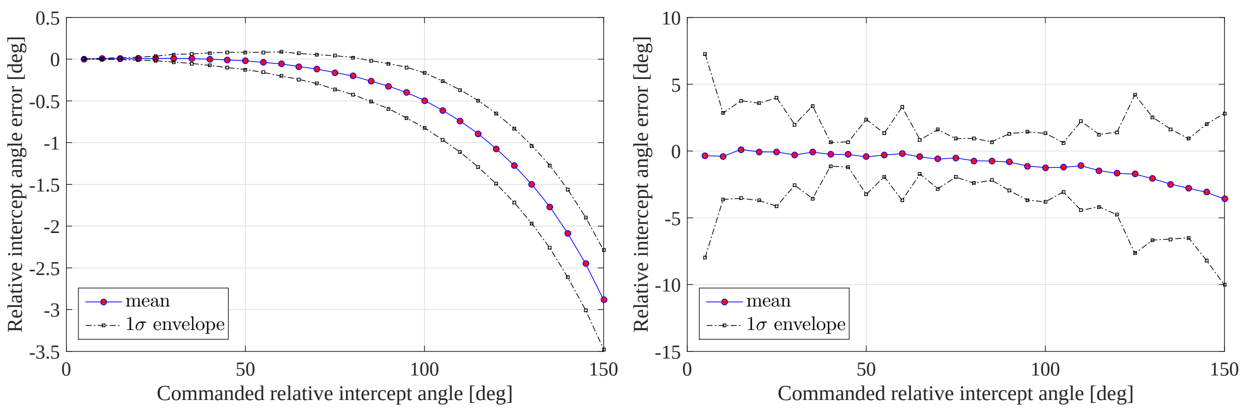
Finally, Fig. 10 depicts the mean and the corresponding 1-sigma envelope of the relative intercept angle error as a function of $\Delta_c \in \mathcal{D}_c$. For perfect information and no acceleration bound case ($u_d^{max} = \infty^\star$), it can be seen from Fig. 10a that as Δ_c becomes larger, the error biases towards negative values (meaning that the achieved relative intercept angle is smaller than the prescribed one) and the error variance increases. Such behavior results from a trade-off in the defenders' cost function formulation, see Eq. (45), which at the same time penalizes the control effort, miss, and the relative intercept angle. Furthermore, the linearity assumptions for larger Δ_c is presumably also less valid. Similar conclusions can be drawn for the imperfect information case ($u_d^{max} = \infty$) from Fig. 10b. The only difference is that the error variance also increases for smaller values of Δ_c. This is understandable as the estimation performance significantly deteriorates for $\Delta_c \to 0$.



(a) Values of $a_d^{max}(95\%)$ as a function of $\Delta_c \in \mathcal{D}_c$.

(b) Values of J_{acc} as a function of $\Delta_c \in \mathcal{D}_c$.

Figure 9: Guidance performance - acceleration requirements for unsaturated case.



(a) $u_d^{max} = \infty^\star$ [g] (perfect states)

(b) $u_d^{max} = \infty$ [g] (estimated states)

Figure 10: Guidance performance - relative intercept angle error as a function of $\Delta_c \in \mathcal{D}_c$.

VI. Conclusions

A cooperative estimation/guidance strategy has been proposed for a team of missiles. An example scenario is considered where these missiles are in a role of defending missiles used to cooperatively intercept an attacking missile homing on to a target aircraft. A new reduced-order estimation scheme based on information sharing to cooperatively estimate the relative state and the unknown parameters of the attacking missile has been proposed under the assumption that only shared LOS angle measurements from the defenders are available. The defenders' guidance exploits an explicit team cooperation to impose relative intercept angle constraints between consecutive defenders and an implicit cooperation of the target aircraft. The cooperation from the target's point of view stems from the fact that the defenders are aware of the evasion strategy of the target and thus can predict the maneuvers it will induce on the homing missile.

Extensive nonlinear simulations revealed that there is a strong influence of the defenders' cooperative guidance on to the estimation performance and vice versa. For a team of two defenders and the target performing a constant turn at 5 [g], it was found that imposing different relative intercept angles lead to distinct effects on the pure estimation and on the intertwined guidance-estimation performance. Small relative intercept angles yield to observability issues. This consequently results in control saturation and severe degradation in the intercept performance. Relative angles ranging from approx. 30 [deg] to approx. 65

[deg] exhibit good estimation as well as guidance performance while maintaining modest maneuverability requirements. Larger intercept angles lead only to negligible improvements in the estimation accuracy, while limiting the defenders' maneuverability worsens the target's protection capabilities for too large intercept angle commands.

The demonstrated capability of the proposed cooperative algorithm can, for carefully selected relative intercept angles, considerably improve the aircraft's survivability from a homing missile, making it possible to design relatively inexpensive defending missiles without advanced sensor systems and large lethal warheads.

Acknowledgments

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Appendix: Defenders' Guidance Parameters

The navigation gains of the defenders' guidance law for a team size of two and $\eta_1 \triangleq 1$ are [34]

$$N_{Z_1}^{u_1} = 3k_{d_1}(t_{d_1m}^{go})^3\alpha_1[t_{d_2m}^{go}V_{d_1}^2C_{22} + 2V_{d_2}^2\eta_2^2C_{21}(2V_{d_1}^2 + \beta_1 t_{d_1m}^{go})]/\Delta_z$$

$$N_{Z_2}^{u_1} = 3k_{d_2}(t_{d_2m}^{go})^2(t_{d_1m}^{go})^2V_{d_1}V_{d_2}\beta_1\alpha_2\eta_2^2(6 - k_{d_1}^2(t_{d_1m}^{go})^3\alpha_1)/\Delta_z$$

$$N_{\Delta Z_3}^{u_1} = 2V_{d_2}^2\beta_1\eta_2^2(t_{d_1m}^{go})C_{21}(k_{d_1}^2(t_{d_1m}^{go})^3\alpha_1 - 6)/\Delta_z$$

$$N_{Z_1}^{u_2} = 3k_{d_1}(t_{d_1m}^{go})^2(t_{d_2m}^{go})^2V_{d_1}V_{d_2}\beta_1\alpha_a(6\eta_2^2 - k_{d_2}^2(t_{d_2m}^{go})^3\alpha_2)/\Delta_z$$

$$N_{Z_2}^{u_2} = 3k_{d_2}(t_{d_2m}^{go})^3\alpha_2[t_{d_1m}^{go}V_{d_2}^2C_{12} + 2V_{d_1}^2C_{11}(2V_{d_2}^2\eta_2^2 + \beta_1 t_{d_2m}^{go})]/\Delta_z$$

$$N_{\Delta Z_3}^{u_2} = 2V_{d_1}^2\beta_1(t_{d_2m}^{go})C_{11}(6\eta_2^2 - k_{d_2}^2(t_{d_2m}^{go})^3\alpha_2)/\Delta_z$$

where

$$\Delta_z = V_{d_2}^2 t_{d_1m}^{go}C_{12} + V_{d_1}^2 t_{d_2m}^{go}C_{11}C_{22} + 4V_{d_1}^2V_{d_2}^2C_{11}C_{21}\eta_2^2$$

$$C_{11} = 3 + k_{d_1}^2(t_{d_1m}^{go})^3\alpha_1, \quad C_{12} = \beta_1\eta_2^2\left(12 + k_{d_1}^2(t_{d_1m}^{go})^3\alpha_1\right),$$

$$C_{21} = 3\eta_2^2 + k_{d_2}^2(t_{d_2m}^{go})^3\alpha_2, \quad C_{22} = \beta_1\left(12\eta_2^2 + k_{d_2}^2(t_{d_2m}^{go})^3\alpha_2\right),$$

$$k_{d_1} = \cos(\gamma_{d_1} - \lambda_{d_1m}), \quad k_{d_2} = \cos(\gamma_{d_2} - \lambda_{d_2m}).$$

The zero-effort-miss distances, Z_1 and Z_2, are

$$Z_1(t) = -V_{\rho d_1m}\dot{\lambda}_{d_1m}(t_{d_1m}^{go})^2 + \int_t^{t_{d_1m}^f}(t_{d_1m}^f - \tau)a_m(\tau)\cos\left(\gamma_m(\tau) + \lambda_{d_1m}(\tau)\right)\mathrm{d}\tau,$$

$$Z_2(t) = -V_{\rho d_2m}\dot{\lambda}_{d_2m}(t_{d_2m}^{go})^2 + \int_t^{t_{d_2m}^f}(t_{d_2m}^f - \tau)a_m(\tau)\cos\left(\gamma_m(\tau) + \lambda_{d_2m}(\tau)\right)\mathrm{d}\tau,$$

whereas the zero-effort flight-path angles, Z_3 and Z_4, are as given in Eq.(47b).

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